Marketing Management Support in Slaughter Pig Production

Ph.D. Thesis

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Preface

This Ph.D. thesis is intended to fulfill the requirements for the Ph.D. degree at The Royal Veterinary and Agricultural University, Denmark. The thesis is the partial result of Dina (Danish Informatics Network in Agricultural Sciences) Research Project no. DSS–APM–3, Delivery Policies in the Slaughter Pig Production. The research was carried out at Department of Animal Science and Animal Health, The Royal Veterinary and Agricultural University and during 9 months in 1994 at Department of Clinical and Population Sciences, College of Veterinary Medicine, University of Minnesota.

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General introduction

In Denmark approximately 19 million pigs are marketed for slaughtering yearly, representing a total value of approximately 14.7 billion DKK to the producers (anonymous, 1995). The majority of these pigs are evaluated and priced based on carcass merits (primarily carcass weight and carcass leanness) of the individual pig. The manager of the individual slaughter pig operation determines when to market individual pigs and groups of pigs. The specific problem of *deciding when to market* (groups of) pigs will in the following be referred to as “the slaughter pig marketing management problem”. How the individual manager solves this problem and determines which pigs to market, differs between operations and managers, but the decisions are in general based on a combination of observations made within the herd, by rules of thumb (like “market the heaviest 20% of the pigs at first marketing from a batch” or “at least 80% of the marketed pigs should be within a given weight range”), tradition, simple calculations and general recommendations concerning threshold weights and batch sizes. In Denmark these recommendations are typically supplied by the Danish Slaughter Houses (see e.g. Rasmussen, 1989; Flemin & Udesen, 1994).

The quantity and quality of herd and animal specific data (e.g. data from the evaluation sheets from the packers) is at present high in most operations and is very likely to increase as new tasks, technologies and techniques evolve, while the costs of acquiring these data will decrease (see e.g. Saatkamp, 1995, for an overview of automatic identification systems and a description of the task of national disease control or Raemakers et al, 1995, for an example of automatic balance weighing of pigs). However, only a minor fraction of these data is at present utilized in a formalized way in managing the marketing of slaughter pigs; No tools for supporting the management of the marketing based on the actual performance of the herd – or “the state of the herd” – have (at least in Denmark) been developed or implemented. Given the complexity of the management problem concerning pricing systems and constraints (see below), given the variation within and between operations and given the simplicity of the management methods presently applied (as discussed above) it is reasonable to assume that at least some managers are able to improve (i.e. increase the return of) their marketing management and consequently there is a potential need of marketing management support tools (MMST); The actual need will depend on the value of the information supplied by the tools and the actual demand, will as discussed below, depend on the expected profitability of using the tools.
The planning of MMST—investments does in principle not differ from traditional production investment planning (i.e. in production facilities and equipment); the expected profitability (i.e. benefits vs. costs) of the investment is the main criterion (Jørgensen, 1992; Verstegen et al., 1995). The expected costs of purchasing and using a MMST (i.e. the costs of acquiring and processing data and of digesting and interpreting information) are in most situations assessed with ease and a high certainty, while the expected benefits are more uncertain and difficult to measure and assess (see e.g. Verstegen, 1995). The output of a MMST is information and the value of this information (i.e. the utility of using the MMST) depends on a set of factors, all (more of less) related to the manager and the management problem: The relevance, the degree of novelty, the accuracy and the timeliness of the information (Bee & Bee, 1990; Kroenke & Hatch, 1994). All four factors are of importance when developing and implementing a MMST, but in particular the requirement of a “high” accuracy might be difficult to fulfil and might partly explain the lack of implemented MMSTs. The main reasons for these difficulties originate from a set of special features of the slaughter pig operations:

- **Biological variance and uncertainty** concerning the “true” state of the operation. Biological and/or stochastic processes and differences in genetic background, health status etc. result in significant variances between individual pigs and between operations. In most situations the manager will, in order to increase the financial outcome, try to reduce the variance between (marketed) pigs within the herd, primarily by selecting and grouping pigs for marketing. Combined with observational errors and missing observations, this discrimination of pigs results in data which are imprecise and biased and which in general can not directly be used to estimate “the true state of the herd”.

- **Operational constraints.** In most situations the pigs are managed and handled in groups or batches, thereby limiting the options of managing and handling individual pigs. The supply of new weaners will normally be fixed (or at least expensive to change) in the short run. In addition a set of constraints may apply directly to the marketing of pigs (e.g. a fixed number of pigs per marketing).

- **The evaluation and pricing system.** In most situations and in order to encourage the production of special kinds of pigs (e.g. lean pigs), premiums will be paid or discounts deducted based on carcass merits. The price paid is not linear in the traits and traditional simple calculations based on expectations of (and ignoring variations on) traits might result in significant errors. In Denmark the same evaluation and pricing system is used by all packers, but in other markets there may be considerable differences between the systems used by different packers (see e.g. Stahl (1991) for a survey of the US market showing a difference in return on identical pigs ranging from $3 to $16); the marketing management problem is in those cases extended from “when to market (groups of) pigs” to “when and how to market (groups of) pigs”.

In order to increase the accuracy and expected benefits, these aspects should be considered (and preferable included) when examining and developing solution methods for the slaughter marketing management problem.

### 1.1 Objectives of the project

The objective of the research project was to investigate and/or develop fundamental and general **models and methods** for solving “the marketing management problem” in general and (if
implemented and used as MMSTs) for supporting the marketing management in the individual slaughter pig operation. In order to increase the accuracy and timeliness of the information produced by the models and methods, high priority was given to the utilization of the (increasing quantity and quality of) herd specific registrations and to the handling of the difficulties imposed by the special features of the slaughter pig operations as discussed above.

It was beyond the scope of the project to develop and implement applicable management support tools or to assess the actual value of using such tools in practice.

1.2 Outline of the thesis

The thesis consists of 4 main chapters (chapter 2 – 5), a general discussion (chapter 6) and summaries (in English, chapter 7 and in Danish, chapter 8).

In chapter 2 general concepts of Information Theory, Computer Based Information Systems (CBIS), Management Information Systems (MIS), Belief Management Systems (BMS) and Decision Support Systems (DSS) are presented and discussed. It is discussed how the task of supporting marketing management might be partitioned into a general task of handling and estimating the belief in the current state of the herd (by the used of a BMS) and a more specific task of supporting the decisions associated with marketing by the use of a DSS and based on the output of the BMS.

In chapter 3 models and methods of a BMS are developed, presented and discussed. The emphasis is put on utilizing herd specific registrations, reducing uncertainty and producing accurate marketing management information. A model for representing the state of the herd is described and used as an interface between the BMS and the DSS.

In chapter 4 models and methods of a DSS are developed, presented and discussed. Optimization methods based on basic production economics and asset replacement theory are applied and different aspects of operational constraints are examined and discussed.

In chapter 5 a specific optimization method used in chapter 4 is presented. The content of the chapter is quite technical and the chapter might be omitted when reading the thesis without losing the overview. In chapter 6 the proposed models and methods are discussed in the light of the overall aim of implementation as a MMST and general limitations, implications, conclusions and future perspectives of the research are outlined.

Chapter 3, 4 and 5 are partly based on the papers Kure (1996), Kure (1997) and Kure (1995) respectively. Chapter 3 and 4 are intended for journal publication.

All models and methods presented in this thesis have been implemented in the Object Oriented Programming Language, C++ and hundreds of objects and thousands of source lines have been written. Neither the process of designing these programs nor the programs themselves are presented in this thesis, but can be supplied from the author on request.

All the four main chapters are selfcontaining expositions that might be read independently of each other.
1.3 References


Basic concepts of Computer Based Information Systems in slaughter pig marketing

2.1 Introduction

The concept and science of Computer–Based Information Systems (CBIS) have evolved from basic automation and data processing systems in the dawn in the 1960s (Ward, 1995) to today’s huge and sophisticated Data Warehouses and Data Mining systems.

In this chapter some basic elements of information theory, management theory and the theory of CBS are described and discussed in the context of slaughter pig marketing managing.

2.2 The need of information

2.2.1 The management cycle

Around 1914 the French management theorist Henri Fayol recognized that all managers perform five major management functions (McLeod, 1995). They:
- Plan what to do (Planning).
- Organize, in order to meet the plans (Organizing).
- Staff the organization with necessary resources (Coordinating).
- Direct the resources to execute the plans (Deciding).
- Control the resources, to keep them on course (Controlling).

This is the classical model of management, which form the fundament of the management cycle as described by e.g. Huirne (1990) and Bee & Bee (1990) in which the 5 components are linked together in order to depict the cyclic and dynamic nature of the management process.

Depending on the management level the importance of these functions will change: At the strategic level (i.e. long term management – from year to year) the emphasis is put on the planning (and organizing) function, while at the tactical level (i.e. middle term management – month to month) it is put on the organizing (and planning) function and at the operational level...
(short term management – day to day) on the coordinating, deciding and controlling functions.

How well the manager will be able to perform/execute these functions/tasks will to some degree depend on the value of the information he receives. This aspect of information is discussed below.

2.2.2 The value of information

A variety of definitions of information have been given, ranging from “the amount of uncertainty that is reduced when a message is received” proposed by Shannon & Weaver (1949, cf. Kroenke & Hatch, 1994) to “information is knowledge derived from data” or “information is data placed within a context” as proposed by e.g. Kroenke & Hatch (1994). The later of these definitions is the most appealing when looking at the common definition of good or valuable information (Bee & Bee, 1990; Kroenke & Hatch, 1994): Good information is characterized by:

- Pertinence (relevance)
- Timeliness (old information is no information)
- Accuracy
- Reduced uncertainty and/or an element of surprise.

However, all these characteristics relate to the context and the needs and objectives of the manager. Valuable information to one manager, might be annoying noise to another – what to some manager, system or organization may serve as information, might serve as simple data to other managers, systems or organizations. This illustrates another important aspect of information: The amount of information may in itself reduce the value of the information; too little or too much information might decrease the value.

To summarize, Bee & Bee (1990) define valuable information as:

“The right and accurate information in the right form at the right time, so enabling the manager effectively and efficiently to do his/her job.”

2.3 Computer–Based Information Systems

An Information System (IS) can be defined technically as a set of interrelated components that collect (or retrieve), process, store and distribute information to support decision making, coordination and control in an organization (Laudon & Laudon, 1995). In addition ISs may support managers and workers in analyzing problems, visualize complex subjects and coherence and create new product and knowledge. A CBIS rely on computers for processing and disseminating information. It should be noted that in some presentations the term Management Information Systems (MIS, see below) are used instead of CBIS (Laudon & Laudon, 1995).

2.3.1 The fundamentals of a CBIS

In general a CBIS can be modeled as a set of sub–systems and information flows as illustrated in fig. 2.1. The five sub–systems that all produce information (output) based on data (input) to support managers might be defined as follows (Laudon & Laudon, 1995; McLeod, 1995):

- TPS/AIS. Transaction Processing Systems/Accounting Information Systems. Basic business systems that serve the operational level of the organization. Performs and records the daily routine transactions necessary to conduct the business. A large volume of data processing (gathering, manipulation, data storage and document preparation) is performed in these
systems. Basic information (summarized data) are produced (scheduled or ad hoc) to be used by managers and other sub–systems; examples are income statements and balance sheets. These systems are the main suppliers of information to the environment.

− **KWS/OAS. Knowledge Work Systems/Office Automation Systems.** Systems that promote the creation of new knowledge and technical expertise are properly integrated into the business and formal and informal systems concerned with the communication of information between people inside and outside the firm. Examples are (for KWS) scientific or engineering design workstations and (for OAS) word processing and desktop publishing.

− **MIS. Management Information Systems.** Systems that serve the managers in executing the functions of planning, decision making and controlling primarily at the tactical level and for problem solving. The information describes the firm in terms of what has happened in the past, what is happening now and what is likely to happen in the future. Most information is in the form of standardized periodic reports, but may be special ad hoc reports. Only on rare occasions the systems will contain/utilize sophisticated mathematical models or statistical techniques.

− **DSS. Decision Support Systems.** Systems (first introduced/defined by Gorry & Morton, 1971, cf. McLeod, 1995) aimed at serving managers in solving particular (decision) problems at any management level. While the MISs are designed for regular and recurring needs, the DSSs are designed for specific tasks and needs. DSS are analytic, dynamic, adaptable and flexible; the user of the system can change/adjust the system as problems changes. DSS are (or might be) complex and may contain sophisticated mathematical models (as e.g. simulation and optimization models known from Operation Research).

− **ESS. Executive Support Systems** (or Executive Information Systems, EIS). Serve executive managers at the strategetical level. ESSs utilize external information (on competitors, new laws etc.) as well as summarized information from MISs and DSSs. They filter, compress and track critical data and give easy (quick and readily) access to information.

It should be noted that in some presentations an additional sub–system is defined as part of the CBIS: **Expert Systems (ES)** or knowledge–based systems, which generally speaking are **Artificial Intelligence** models/computer programs that attempts to represent the knowledge of human experts. In addition to this, two new categories of systems have been introduced recently: **Data**

![Diagram](https://via.placeholder.com/150)

**Figure 2.1.** The Computer based information system (CBIS). Arrows represent information flows: Information (output) produced by one sub–system is data (input) for another sub–system. See text for explanation of symbols.

(Laudon & Laudon, 1995)
Warehouses and Data Mining in which huge amounts of data are stored, organized and analysed using advanced statistical methods and artificial intelligence (Bracket, 1996).

2.4 A computer based slaughter pig marketing information system

The presented model of a CBIS in fig. 2.1 is a general model that may apply to any category of firms. In the following the special features of the slaughter pig operations and in particular the marketing function will be discussed and a basic model of a computer based slaughter pig marketing information system presented.

2.4.1 Coping with biological variance and uncertainty

As demonstrated in chapter 3 and 4 pigs are not equal; even within the same slaughter pig operation the pigs and the traits of the pigs may vary considerably. In addition, some traits can be difficult or even impossible to observe and measure, implying uncertainty concerning the “true” value of the traits – the recorded data “do not tell the whole truth directly”. In order to support the marketing management decisions, tools for representing and handling this variance and uncertainty have to be applied (see chapter DSS). However, the basic model of the CBISs as defined above does not directly include such tool and in the following a revised/extended CBIS model is presented.

2.4.2 The model

The model presented here is based on the conceptual outline of a slaughter pig simulation model developed and described by Jørgensen & Kristensen (1995). The simulation model contains a simple CBIS containing two subsystems: A DSS and a Belief Management System (BMS). The BMS models the manager’s beliefs in the state of the system (the pigs and traits of the pigs) and the system dynamically process new data/information into current/updated beliefs. The term Belief is inspired by the terminology of Bayesian Networks (a special class of ES – see e.g. Jensen, 1996) and an implementation of the BMS might be based on a Bayesian Network. As the BMS can (and normally will) be based on sophisticated statistical methods it does not easily fit into the CBIS as defined above. However, it might be characterized/defined as an advanced AIS or MIS; The purpose of the system is to produce management information to be used directly by the manager or as an input for a DSS.
A revised version of the model proposed by Jørgensen & Kristensen (1995) is shown graphically in fig. 2.2. A central element in the model is the manager who receive information from 4 major sources: Directly from the production system (raw data), from the environment (external information), from the BMS (beliefs combined with ordinary information from the AIS/MIS) and from the DSS (decision aid). Based on this information the manager supplies decisions/plans to the production system and revised objectives, plans, knowledge and beliefs to the BMS. Beliefs produced by the BMS serve as an interface between the BMS and one or several DSSs; the BMS produces general information that may be utilized by different DSSs.

This basic model (as illustrated in fig. 2.2) form the conceptual framework of the models presented in chapter 3 and 4 and the two major sub–systems – the BMS and the DSS – constitute a Marketing Management Support Tool as defined in the general introduction (chapter 1).

2.5 References


A slaughter pig marketing belief management system

Abstract. Data from most slaughter pig operations will normally be highly biased due to selection (censoring) of individual pigs prior to the termination of the individual batch. An animal growth model based on multi variate normal distributions is introduced and used as representation of the “state of the herd”. Methods for reducing the bias and for updating the “Belief in the state of the herd” are presented. The methods are based on the EM–algorithm and other methods for bias reduction and on the Kalman filter for belief updating. Models are tested based on simulated data. Assuming that pigs are identified by batch–#, the results show good performance of the methods even in the case of very sparse data–sets. One of the main features of the Kalman filter – the quick adaption to changes in the system modeled – is demonstrated by examples.

3.1 Introduction

Knowledge of or a realistic Belief in) the current and expected future “state of the slaughter pig finishing operation” is crucial in efficient management in a modern slaughter pig production. At present a large quantity of herd specific data is generated internally (i.e. at the farm) and externally (e.g. at the slaughter house) and the quantity and quality of these herd (and animal) specific data is likely to increase in the future as new technologies evolve (see e.g. Geers, 1994 or Saatkamp, 1996, for a current status on identification and recording systems in pig production or Ramaekers et al., 1995 who present a new efficient method for automatic estimation/registration of individual live weights). However, tools for managing and processing this wealth of farm specific data into (for the marketing management) valuable (i.e. precise, relevant and topical) information are not yet developed and/or implemented in practice.
One of the objectives of a Danish research project initiated in 1992 was to develop such tools and the main results of that part of the project are presented in this paper, starting with a short general description of the slaughter pig finishing operation and the problems associated with handling and utilizing herd specific data.

3.1.1 The slaughter pig finishing operation

The *slaughter pig finishing operation* is assumed logical and operational partitioned according to a physical sectioning of the operation facilities (e.g. confinements). The group of pigs in a *section* is defined as a *batch*; all pigs in a batch must be marketed (or moved to another section) before insertion of a new batch (group of weaners) is possible, but pigs in a batch might be marketed individually. The period from the first to the last feasible marketing from a batch is partitioned into *marketing stages*, each stage representing the moment (e.g. a certain day of the week) at which the manager has the option (but not the obligation) to market pigs from the batch. Usually the duration of the marketing of all pigs in a batch will be no more than 4 weeks (i.e. 5 marketings). The last marketing from a batch (i.e. marketing of all remaining pigs in the section) is defined as the *terminal marketing* (the batch is terminated) and it is succeeded by an insertion of new pigs or weaners. The duration of the insertion may range from a single day to weeks resulting in differences in pigs’ physiological age within a batch. In most situations the pigs/weaners will at the time of insertion be preselected and grouped within the section according to their size in order to simplify the management (e.g. feeding and marketing). However, in this presentation there will be no conceptually distinction between this pre–selection at insertion time and the selection at marketing time.

Pigs are marketed to one or several *packers* (slaughter houses) and the prices paid for pigs are based on a packer–specific *pricing system*. The pricing system will normally favorite pigs within certain *carcass weight* and *leanness* intervals. The results (e.g. leanness, carcass weights and financial outcomes) of a marketing are reported on an *evaluation sheet*.

Based on her/his belief in the herd and in individual pigs the manager will decide when and how (i.e. to which packer) to market the pigs. The manager can market all the pigs in a batch at the same time or she/he can select individual pigs from the batches in order to reduce the variance on the pigs and consequently increase the number of pigs paid the highest price per kg. As illustrated in table 3.1 the variance is drastically reduced from 49 kg$^2$ without selection and marketing after 16 weeks to approximately 4 kg$^2$ when selection based on observed live weight and marketing at all 5 ages is applied. The problem of selection of pigs for marketing and

<table>
<thead>
<tr>
<th>Age</th>
<th>Selection on live weight, W</th>
<th>No selection</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>E (CW)</td>
<td>V (CW)</td>
</tr>
<tr>
<td>12</td>
<td>76.63</td>
<td>5.02</td>
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<td>13</td>
<td>76.17</td>
<td>2.71</td>
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<td>76.17</td>
<td>2.46</td>
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<td>15</td>
<td>75.66</td>
<td>3.21</td>
</tr>
<tr>
<td>16</td>
<td>72.79</td>
<td>12.47</td>
</tr>
</tbody>
</table>

1) Weeks since entering the grower finishing operation (at a weight of 25 kg).

2) Fraction of selected pigs out of total pigs.
3.1 Introduction

selection criteria in general are discussed in details in chapter 4.

Several traits of the pigs can be relevant to the manager when planning and executing the marketing of pigs and should therefore be included in the representation of the belief in the “state of the pigs in the herd” or just the belief in the herd. Carcass weight and leanness are as indicated above the economically most important traits, but as these traits will not be exposed until at slaughter, they are not known prior to marketing and therefore covariates (i.e. other correlated traits) have to be observed and used for the decision making. Usually live weight and/or age are used for this purpose, but other traits like back fat thickness (see e.g. Smith et al., 1992) or physical dimensions (see e.g. van der Stuyft et al., 1991) may be used. Several measuring methods with different precision or observational errors (see e.g. Jørgensen, 1993 for a comparison of different methods for measuring live weight) like visual or balance weight assessment can be used for measuring the value of these traits.

3.1.2 Incomplete, biased and imprecise data

Farm specific and marketing management relevant data are supplied from several external (e.g. kill sheet data) as well as internal (e.g. automatic feed consumption measurements or live weight balance weights performed by the staff at the time of marketing) sources, but, as discussed below, it is difficult to combine data from different sources. Data may be recorded continuously (e.g. feed consumption and in special situations live weights), but most data are recorded discretely (e.g. once a week).

In many cases the purpose of acquirering the data is not directly to support the marketing management, but even in those cases the utility of the data may be of great magnitude with respect to the marketing management. An example of this kind of data are the evaluation sheet data, which are acquired in order to price individual pigs, but can be very valuable in the management of the marketing.

Data can be partitioned into two groups according the aggregation level:

- Individual data which are data registered on individuals.
- Aggregate data which are registrations on groups of individuals (e.g. averages) or aggregated individual data.

In most situations the data will be incomplete (i.e. missing registrations at the individual level), biased (i.e. systematically missing registrations at the aggregate level) and/or imprecise (at both levels). These errors in the data are in general caused by:

- Latent traits. Traits which are not exposed until at slaughter. These traits (e.g. carcass weight and carcass leanness) are only observed and registered once in the life time of the individual pig and the values of the traits are unknown, or at least uncertain, at the time of selection of that pig for marketing. Data made on latent traits will when combined with selection (see below) be biased.

- Selection (or censoring). Individual pigs are selected for marketing primarily in order to reduce the average variation on financially important traits (and increase the financial outcome) and secondarily in order to fulfill other objectives such as a constant number of pigs marketed per week or a reduced stocking density in the confinements. As a consequence of this selection, aggregate data are biased and individual registrations incomplete (i.e. missing longitudinal data – some individuals will be observed and registered over a shorter period than others). Pigs are selected based on observable traits,
which normally are highly correlated with the latent traits.

It should be noted that missing observations are not occurring at random; the likelihood of missing observations on an individual depends heavily on the traits of the individual and consequently most general approaches to handling missing observations are non-applicable (Little & Rubin, 1987).

- **Missing identification** of individuals or groups of individuals. In most situations it is not possible to identify the individual pig within the finishing operation (and therefore in general it does not serve any purpose to register observations on individuals) and combining data from different sources (e.g. kill sheet data and on farm balance weights) directly are not possible. In addition pigs from different batches may be mixed during the shipping to the packing which makes identification of separate batches difficult. However, in the remaining of this paper it is assumed that the batches are evaluated separately or that pigs are identified by batch—#.

- **Observational errors.** All traits are possibly exposed to observational errors, which are caused by: Errors in the definition and the measuring of traits, errors in the identification of individuals or groups, timelags and other errors including labor and machinery malfunction.

- **Other errors.** Machinery failure, labor shortage, etc.

The two terms “observation” and “registration” are in this presentation used interchangeably; “observation” is used as a pseudonym of “registration”.

In most slaughter pig finishing operations it is financially beneficial to select individual pigs for marketing (see chapter 4 for calculations of the benefit of selection in the more general case) and in the remainder of this presentation piggies are therefore assumed to be selected and hence data assumed to be biased/incomplete, with the situation of unbiased data as a simple special case. In addition there will in many operations be no longitudinal data (e.g. evaluations sheet data are the only data available) and hence the existence of longitudinal data will be no prerequisite in this presentation.

### 3.1.3 Modeling and estimating the “Belief in the state of the herd”

The definition and representation of “the state of the herd” will depend on the actual context and objectives and hence in this presentation “the state of the herd” is defined and represented based on the main underlying problem: Efficient slaughter pig marketing management. In chapter 4 it is demonstrated that optimization of marketing management is, at any given marketing stage, based on knowledge of

(i) the value (or state) of traits of the individual pig at the current stage and

(ii) the change in the states of the traits (e.g. growth) between the current and the next stage(s) (i.e. change = value at next stage – value at current stage). In principle there are two reasons for this change: (a) *Growth of animals* and (b) *system changes* (i.e. in the genetic potential and in the environment).

This change in states of traits is traditionally modeled by some kind of functional relationship and these *animal growth models* can be partitioned into two separate groups (Oltjen, 1992; Black, 1995):

- *Explanatory, mechanistic models and*
3.1 Introduction

- **predictive, empirical models.**

Mechanistic models are detailed models based on knowledge of the underlying physiological processes and laws of physics and chemistry. The models are in most situations deterministic (i.e. only expectations are represented in the models) and normally a large set of model parameters are very difficult to interpret and estimate (see e.g. Black *et al.*, 1986; Moughan *et al.*, 1987; Moughan *et al.*, 1995; Black, 1995 for examples on mechanistic growth models). Mechanistic models are in general not well suited for management and control, but Oltjen (1992) demonstrated a way of extending mechanistic models with an empiric element in order to calibrate a mechanistic model according to a few empiric data (which are insufficient in re-estimating parameters of the mechanistic model). In general it is difficult to separate the two types of models and several models can be described as mixtures of the two (see e.g. Jørgensen, 1993 in which a mechanistic model based on the Gompertz growth curve is extended with stochasticity). Mechanistic models do normally not include features for modeling/estimating **system changes** – i.e. the second element of changes in traits as discussed above (Oltjen, 1992 and Jørgensen, 1993 are exceptions).

Empirical growth models are statistical models describing the relationships between traits/variables over time without describing underlying processes. The models are stochastic (i.e. variation on the traits as well as expectations are modeled) and since variables in most cases are easy to observe and since the number of parameters in the models is low in relation to the number of observations, parameters are more easily estimated than in the case of mechanistic models. A main and very general and well established class of empirical growth models are based on the work of Potthoff & Roy (1964): Growth Curve Models or Generalized Multi variate Analysis of Variance Models (see e.g. von Rosen, 1991 for a review). These models are very robust, but in general they do not accept incomplete data (Liski, 1985; Kanda, 1994). However, Liski applied a **data augmentation method** (the EM algorithm, Dempster *et al.*, 1977; Little & Rubin, 1987; Tanner, 1991) in order to generate the (randomly) missing observations. Empirical models normally include features for modeling/estimating **system changes**.

The general problem of statistical inference with missing data is a classic statistical problem that has been examined by a wide range of scientists (see e.g. Little and Rubin (1987) for an introduction and overview). In most cases the data are assumed to be missing at random (which is certainly not a reasonable assumption in the situation of slaughter pig marketing), but methods for handling special patterns or mechanisms of missing data (e.g. grouped or censored data) have been developed (Little & Rubin, 1987).

Aggregate data are not as informative as individual data (i.e. aggregate data may be derived from individual data but not vice versa) and as indicated in the previous section greater problems associated with utilizing these data should be expected. However, in some cases a large amount of aggregate data is produced within the slaughter pig operations and methods for utilizing these data can prove beneficial when applied.

The objective of this study was to develop (and examine) simple, general, reliable and applicable methods (and in particular methods based on the EM algorithm) for dynamically transforming (aggregate and individual; incomplete and/or biased) herd specific data into precise, topical and marketing management relevant information. As demonstrated in chapter 4 correct estimates of expected growth are of greater importance than correct estimates of (co)variances on growth and therefore the emphasis is put on estimating correct expectations.
The methods should form the fundament of a Belief Management System (BMS) as defined in chapter 2 and by Jørgensen & Kristensen (1995). The BMS is intended for integration in a Marketing Management Support Tool (see chapter 1) as part of a larger Management Information System (see chapter 2) and it is intended to serve as a direct preprocessor of the Decision Support System specified in chapter 2 and defined in chapter 4.

The issue of acquiring additional information (i.e. techniques/sources and costs/benefits) is not examined in this context.

The purpose of this paper is to present and discuss the methods developed and/or used.

### 3.2 Models and methods

The updating of the belief in the "state of the herd" is in this presentation partitioned into two steps:

- A data augmentation and bias reduction step and
- a belief updating step.

Different methods for reducing the bias are presented here, while same representation (multivariate normal distributions) and same belief updating method (Dynamic Linear Models, DLM or Kalman filtering) of the "state of the herd" is used for all bias reduction methods.

#### 3.2.1 The animal growth model

As demonstrated in the introduction and in chapter 4 the marketing of pigs is only possible at certain times (e.g. at a certain day of the week) and only the belief in the value of the traits at the time of marketing and for some traits at the time of deciding which pigs to market is of interest to the marketing management problem and should be represented in the model. Consequently (and in contrast to traditional growth models as discussed in the introduction) there is no need for a continuous representation of the traits of the pigs and in this presentation a discrete and rather simple representation of growth is used: Traits (and growth) of pigs at different marketing stages are modeled directly by a multivariate normal distribution as defined below. In the following it assumed that the batch \# of each pig is known at any time in the process from insertion to slaughter.

Let $M$ be the number of animal traits included in the model and $N$ be the last feasible marketing stage of a batch. With the option of marketing once or not at all in a week and with a 5 weeks duration of the marketing of a batch, $N = 5$.

Let the true traits (i.e. values not influenced by measuring or observational errors) of a pig at a given marketing stage (or marketing age), $n$ and at a given time, $t$ be represented by a $M$-dimensional vector, $X_{nt}$. The traits are assumed multivariate normally distributed:

$$X_{nt} \sim \mathcal{N}(m_{nt}, C_{nt}), \quad n = 1, \ldots, N$$

where $m_{nt}$ is a $(M \times 1)$ vector of means and $C_{nt}$ is a $(M \times M)$ matrix of variances and covariances.

The time, $t$, will later be referred to as the updating step (see section 3.2.3: Belief updating – the
Figure 3.1. The animal growth model. Mean growth (at a decreasing rate) and distribution of live weight at 5 different marketing stages. Pigs with a weight above the threshold weight are (except for the last stage at which all remaining pigs are marketed) selected for marketing. Abrupted straight lines have been applied to illustrate the blurring truncations of the left tail of these conditional distributions; i.e. truncation at previous stage is, unless 100% dependence between previous and current stage, blurring at current stage.

Kalman filter.

The set of variables $X_{nt}, n = 1, \ldots, N$ are merged into a single variable $X_t$, representing a pig’s traits at all $N$ marketing stages at a given time $t$:

$$X_t \sim N(m_t, C_t) \tag{3.1}$$

where:

$$X_t = \begin{bmatrix} X_{1,t} \\ X_{2,t} \\ \vdots \\ X_{N,t} \end{bmatrix}, \quad m_t = \begin{bmatrix} m_{1,t} \\ m_{2,t} \\ \vdots \\ m_{N,t} \end{bmatrix}, \quad C_t = \begin{bmatrix} C_{1,t} & \text{sym} \\ C_{2,1,t} & C_{2,t} \\ \vdots & \vdots \\ C_{N,1,t} & C_{N,2,t} & \ldots & C_{N,t} \end{bmatrix}$$

The covariance between the partial distributions ($C_{ij,t}, i \neq j$) express the correlation between traits over marketing stages, $n$. In the remainder of this presentation the time suffix, $t$, is assumed implicitly given and therefore in most cases omitted in the notation used.

Two traits are included in the model used in this presentation (i.e. $M = 2$): Live weight ($W_n$) and carcass weight ($CW_n$) i.e. $X_n = [W_n, CW_n]^T$, examples are shown in the appendix. It is assumed that pigs are individually identified within the slaughter pig operation as well as at the packer and that there exists a mapping between these to identifications. This assumption will later be relaxed. Graphically the animal model can be (partially, i.e. for a single trait) represented as shown in fig.
3.1. The mean growth curve has at each marketing stage been extended with a graph representing the distribution of weight at that particular stage. The shaded areas of the distributions illustrate, as the simulated data in table 3.1, the effect of selecting pigs for marketing: Reduction of variance.

The conditional distribution (or the posterior distribution) of the true traits $X$ given a specific observation (on any subset of the traits) and given a measuring or observational error is derived as follows: Let $X_2$ represent the observed subset of traits of $X$, and let $X_1$ represent the not observed traits. Then $X$ is partitioned into represented as:

$$ X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \sim N \left( \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}, \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \right) $$

The observational errors on the observed traits, $X_2$, are represented by a vector, $\varepsilon_2$, of random variables which are assumed independent of $X_1$, mutually independent and identically multivariate normally distributed with zero mean: $\varepsilon_2 \sim N \left( 0, E_2 \right)$.

The observed traits (e.g. the observed live weight) are then normally distributed and defined by the model:

$$ X' = \begin{bmatrix} X_1' \\ X_2' \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_2 \end{bmatrix} \sim N \left( \begin{bmatrix} m_1' \\ m_2' \end{bmatrix}, \begin{bmatrix} C_{11}' & C_{12}' \\ C_{21}' & C_{22}' \end{bmatrix} \right) $$

The distribution of $X$ given an observation $x_2'$ of $X_2'$ and given the observational error $\varepsilon_2$ is then given by:

$$ X | X_2' = x_2', \varepsilon_2 \sim N \left( m_1', \begin{bmatrix} C_{11}' & C_{12}' \\ C_{21}' & C_{22}' \end{bmatrix} \right) $$

(3.2)

where:

$$ C_{E} = C_{22} + E_2 \quad (a) $$

$$ m_i' = m_i + C_{2i} C_{E}^{-1} (x_2' - m_2), \quad i = 1, 2 \quad (b) $$

$$ C_{ij}' = C_{ij} - C_{2i} C_{E}^{-1} C_{2j} \quad i, j = 1, 2 \quad (c) $$

3.2.2 Herd model

If the traits of all pigs in a herd are identically normally distributed the “state of the herd” (or the belief in the herd) can be described by the animal model presented above. However, in most cases
the traits are not identically normally distributed; the pigs differ due to differences in genetic line, gender, feed diet etc. or due to selection based on e.g. age or size. In these situations it is possible to categorize or partition the pigs in a given herd into a set of logical groups in such a way that for each logical group the traits of the (unselected) pigs in that group will be (approximately) identically normally distributed. The partitioning can be based on gender, genetic line or feed diet, but several other criteria might be applied in order to satisfy this normality restriction.

The herd is represented or modeled as a set (of size $L$) of sub–models, each representing a logical group. Any pig is member of one and only one logical group, but the pigs in a batch can be member of different logical groups.

The number of logical groups in a herd can vary over time and in general there is no upper limit on the number of logical groups. In the case of a very homogeneous herd with no significant differences between sections (or groups or pens or pigs) a model with one (or three – one for females, one for castrates and one for males) logical groups might appear sufficient.

Each individual logical group, $I$, is represented by an instance, $X_I \sim N (\mu_I, C_I)$ of the animal model presented above and the main objective presented in the introduction (i.e. to generate marketing management information) reformulated as: For each logical group, $I$, estimate the parameters of the distribution, $X_I$, based on the current knowledge of the logical group and on observations made on individuals belonging to the group.

The variance $E_I$ on samples from a logical group $I$ is given by:

$$E_I = C_I + E \quad (3.3)$$

where $C_I$ is the within–group variance and $E$ is the variance on the observational error (with zero mean) of an individual (i.e. the measuring error).

The initial prior distribution, $X_{I_0} \sim N (\mu_{I_0}, C_{I_0})$ of the animal growth model is defined as the distribution prior to any observations.

### 3.2.3 Belief updating – the Kalman filter

A steady Dynamic Linear Model (DLM, see e.g. Harrison and Stevens, 1976; West and Harrison, 1989), also known as Kalman filtering (Kalman, 1960) is used for data filtering/updating the beliefs in the logical groups of the herd – the belief in the “group effects” of the logical groups. The Steady DLM is a simple polynomial DLM of zero degrees:

$$y_t = \theta_t + v_t, \quad v_t \sim N(0,V_v) \quad (a)$$

$$\theta_t = \theta_{t-1} + w_t, \quad w_t \sim N(0,W_{\theta}) \quad (b)$$

where (a) is the observation equation and (b) the system equation. $W_{\theta}$ is the system variance (or disturbance variance) while $V_v$ is the observational variance (or observation noise). Logical groups are assumed independent and they are handled/updated independently.

Given an initial prior distribution $\theta_{I_0} \sim N (\mu_{I_0}, M_{I_0})$ (where $M_{I_0} = \pi C_{I_0}$ and $\pi$ is a predefined constant) of the expectation of the logical group, $I$, and a set of prior information $D_{I_0} = \{y_{I_0}, \sigma_{y_{I_0}}\}$, where $y_{I_0}$ is the observation at time (or updating step) $t$ and $D_{I_0}$ is the initial information, the posterior distribution of the process parameters at step $t$, $(\theta_t | D_t) \sim N(m_{\theta}, M_{\theta})$ may be derived
recursively using a Bayesian or Kalman filter approach (Kalman, 1963; Harrison and Stevens, 1976) as defined by the updating recurrence relationships in table 3.2.

**Table 3.2. Updating recurrence equations of the Steady DLM.** *(Harrison & Stevens, 1976)*

<table>
<thead>
<tr>
<th>Let</th>
<th>Then</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{y} = m_{t-1} )</td>
<td>( m_t = m_{t-1} + Ae )</td>
</tr>
<tr>
<td>( e = y_t - \hat{y} )</td>
<td>( M_t = R - A\hat{y}A^T )</td>
</tr>
<tr>
<td>( R = M_{t-1} + W_t )</td>
<td>( \hat{Y} = R + V_t )</td>
</tr>
<tr>
<td>( A = R(\hat{Y})^{-1} )</td>
<td>( y_t = y_t )</td>
</tr>
</tbody>
</table>

In the model used in this paper \( y_t \) is the “unbiased complete” sample mean of the \( n_t \) individuals observed at time or updating step \( t \) while \( V_t \) is the variance on the observational error on this sample mean. How to eliminate bias and augment data (i.e. generate missing data) in order to assess \( y_t \) and \( V_t \) based on actual data is discussed below in the section: *Bias reduction*. It should be noted that in many practical applications of the Kalman filter, \( V_t \) (and \( W_t \)) are not estimated based on empirical data, but set explicitly by the model user based on model experiments and the intended use of the model (West & Harrison, 1989).

The Kalman filter is used to dynamically update the belief in the expectations, while other (and more simple) methods are applied in order to estimate/update the belief in the variances (i.e. estimates of \( C_t \) in the animal model).

In this presentation a constant decay of information is assumed and modeled as:

\[
W_t = M_{t-1} \frac{(1-\delta_t)}{\delta_t}
\]

where \( \delta_t \) is the variance discount factor \((0<\delta_t<1)\). A large value (e.g. 0.9) of \( \delta_t \) will lead to a little system variance, \( W_t \) and little adaption to observations, while a small value (e.g. 0.001) will lead to a large system variance and hence rapid adaption to new observations.

Mean and variance on the one–step ahead process parameter forecast is given by:

\[
\hat{m}_{t+1} = m_t \quad \hat{M}_{t+1} = M_t + W_{t+1}
\]

As suggested by West and Harrison (1989) this 1 step–ahead estimate can be used for simple extrapolation into the future, determining a constant step–ahead evolution variance matrix:

\[
\text{Var}(w_{t+k} | D_t) = W_{t+1}(k) = W_{t+1}
\]
implying that:

\[ \hat{\mathbf{M}}_{t+k} = \mathbf{M}_t + kW_{t+1} \]

Mean and variance of the \( k \) step-ahead observation forecast are given by:

\[ \hat{\mathbf{y}}_{t+k} = \hat{\mathbf{m}}_{t+k} \]
\[ \hat{\mathbf{Y}}_{t+k} = \hat{\mathbf{M}}_{t+k} + \mathbf{V}_{t+k} \]

3.2.3.1 Model intervention

External information or observation on other traits/variables than those represented in the model can (prior to actually observing changes in the traits modeled) cause changes in the managers belief in the “state of the herd”. These changes are in the model handled simply by changing the prior distributions. The mean \( \mathbf{m}_t \) is changed according to the new expectations (if any), while the covariance \( \mathbf{M}_t \) is changed according to the uncertainty of the belief and/or the desired adaption rate to new observation – a larger (co)variance implies a quicker adaption to new observation.

3.2.4 Bias reduction

For each of the two aggregation level of observations different bias reduction methods are applied:

– Truncated mean transformation, which apply in the situation of aggregate data (i.e. sample means) and

– Data augmentation, which apply in the situation of observations on individuals.

3.2.4.1 Aggregate data – truncated mean transformation, TMT

The highly biased aggregate observations (e.g. the means in table 3.1) made in the finishing operation are in general difficult to utilize in estimating the “state of the herd”; In most situations the manager will try to market as many pigs as possible within a narrow target interval, implying that the mean on selected pigs will be quite unaffected by the actual “state of the herd”. However, the actual number of selected and marketed pigs can reveal valuable information, which combined with knowledge of the selection criterion and assumptions concerning variances and correlations can be utilized. Pigs are in the following assumed selected based on a threshold weight \( T_x \) on trait \( X \) at the previous stage and based on a threshold weight \( T_y \) on trait \( Y \) (same trait as \( X \), but at the succeeding stage) at the current stage. Neither \( X \) nor \( Y \) are necessarily registered. The expectation of a correlated and observed trait \( Z \) (which, when observing \( Y \) can be the same trait as \( Y \)) is approximated based on the correlations between \( Z \) and \( X \) (\( \rho_{xz} \)) and between \( Z \) and \( Y \) (\( \rho_{zy} \)). Some experimentation has resulted in the following approximate relationship:
where \( F_z(T_1) = F_{X}(T_1) \), \( F_z(T_2) = F_{Y}(T_1) \) and \( F \) is the density mass function.

E \((Z)\) is then found iteratively by comparing the approximated value from (3.5) and the actual observed value.

At the first stage there will be no selection prior to current stage and at the last marketing stage no pigs will be selected at current stage. In those special cases (3.5) is simply reduced to not include any elements containing \( X \) and \( Y \) respectively.

Now let \( \tilde{y}_{it} \) represent the vector of the \( M_i \) observed traits at time \( t \) (i.e. \( M_i \leq N_i Z \)) and let \( \tilde{y}_{it} \) represent the corresponding unbiased observations (i.e. (3.5) applied individually to each trait). Then \( y_i \) and \( V_i \) in (3.4) are given by the conditional (i.e. (3.2)):

\[
\begin{align*}
y_{it} &= E(X_{it}|\tilde{y}_{it}, \tilde{E}_{ii}) \\
V_{it} &= V(X_{it}|\tilde{y}_{it}, \tilde{E}_{ii}) \\
\tilde{E}_{iij} &= \frac{E_{ij}}{\sqrt{n_{ij}}} \quad i,j = 1,\ldots,M_i
\end{align*}
\]

where \( E(X_{it}) = m_{i-1}, \)
\( V(X_{it}) = C_{it}, \) the (co)variance of the initial prior,
\( \tilde{E}_{iij} \) are the elements of \( \tilde{E}_{ii}, \) the variance on the observed means,
\( E_{ij} \) are the elements of \( E_i \) in (3.3) and
\( n_{ij}, n_{ij} \) are the number of observations on trait \( i \) and \( j \) respectively.

In fig. 3.2 approximated values (i.e. from (3.5)) have been compared with estimated (true)

\[ \text{corr} (Z, X) = 0.97 \]
Sample size = 200,000

\[ \text{corr} (Z, Y) \]

**Figure 3.2.** Comparison of estimated (true) conditional means and means approximated by the “truncated mean transformation method” based on aggregate data.
values for different values of the unknown correlation between \( Z \) and \( Y \) and with perfect information on all other model parameters except means (parameters from “the perfect initial guess” and with \( X = W_3, Y = W_4 \) and \( Z = CW_4 \) as defined in Appendix 3.A, i.e. \( M = 1, N = 2 \)). As demonstrated in fig. 3.2, not all bias has been eliminated, but for \( \rho_{ZX} \approx \rho_{ZT} \) (which should be expected in most situations) the method appears to be quite precise. The predicting capabilities, the precision of the method and the sensitivity to errors in assumptions (variances and correlations) will be examined in more details in the results section.

The “truncated mean transformation” updating method is summarized as follows: After each marketing (i.e. at each updating step of the Kalman filter, e.g. once a week), calculate for each logical group the unbiased means of the traits of the marketed pigs, based on the observed biased means and by applying (3.5) iteratively. These unbiased means are calculated for each marketing stage separately based on all pigs (belonging to that logical group) marketed at that stage. The “unbiased expected observed mean” (i.e. of all variables – observed as well as not observed) and the corresponding variance to be used in the Kalman filter (3.4) are then given by the conditional (3.6).

### 3.2.4.2 Individual data – the basic EM algorithm, EMI

The \textit{EM algorithm} is a method for estimating model parameters from incomplete observations (see e.g. Dempster et al, 1977; Little & Rubin, 1987; Tanner, 1993). The basic idea of the method is to augment the data by generating the missing (not observed) data based on the current “guess” on the value of the model parameters and the observed data (i.e. the \textit{expectations} of missing data given the observed data and the current guess of model parameters – the E–step). Model parameter estimates are then (re)calculated (parameter likelihood \textit{maximization} – the M–step) based on this “complete” data set and the two steps are repeated until convergence. In the method implemented here the E–step has been extended with Monte Carlo simulations in order to include conditional variances and avoid numerical instability when calculating Maximum Likelihood estimates (see below).

The EM algorithm with simulations is applied as follows:

For each terminated batch and each logical group, \( I \) calculate the observed mean, \( y_i \) and the variance on the mean \( V_y \) as follows.

\( a. \) The E–step:

For each individual, \( i \), calculate the conditional distribution (3.2) of traits, \( X^{(k)}_i \) at the \( k \)th iteration given the observations, \( y_i \), the observational errors, \( \varepsilon_i \) and the prior, \( X^{(k-1)} \):

\[
X^{(k)}_i = X^{(k-1)} | y_i, \varepsilon_i, i = 1, \ldots, n_i
\]

and generate \( Z^{(k)} \) independent and identically distributed samples from that conditional:

\[
(x_z | \varepsilon_i) \sim f_{X^{(k)}_i | \varepsilon_i}(x_z), \quad z = 1, \ldots, Z^{(k)}
\]

\( b. \) The M–step:

The mean and covariance of the posterior \( (X^{(k)} = X^{(k-1)} | x_z, i = 1, \ldots, n_i, z = 1, \ldots, Z^{(k)}) \) are then
given by the Maximum Likelihood (ML) estimates $\mathbf{m}^{(k)}$ and $\mathbf{C}^{(k)}$: 

$$
\mathbf{m}^{(k)} = \frac{\mathbf{S}^{(k)}}{n^{(k)}} \quad \mathbf{C}^{(k)} = \frac{(\mathbf{M}^{(k)} - \mathbf{S}^{(k)}\mathbf{S}^{(k)\top}/n^{(k)})/n^{(k)}}
$$

(3.8)

where:

$$
n^{(k)} = n_{u} \cdot Z^{(k)} \\
\mathbf{M}^{(k)} = \sum_{i=1}^{n_{u}} \sum_{z=1}^{n_{u}} \mathbf{x}_{iz} \cdot \mathbf{x}_{iz}^{\top} \\
\mathbf{S}^{(k)} = \sum_{i=1}^{n_{u}} \sum_{z=1}^{n_{u}} \mathbf{x}_{iz}
$$

Step $a$. and $b$. are repeated with $\mathbf{X}^{(k)}$ as the new prior and $k = k + 1$ until $\| \mathbf{m}^{(k)} - \mathbf{m}^{(k-1)} \|_2 < \epsilon$, where $\epsilon$ is a predefined lower scalar limit. The “observed unbiased mean”, $\mathbf{y}_{\bar{u}}$ which is used in the Kalman Filter (3.4) is then given by $\mathbf{m}^{(k)}$ and the variance on the mean, $\mathbf{V}_{\bar{u}}$, by $\mathbf{C}^{(k)}/n_{u}$, where $K$ is the last iteration step. In the first iteration (i.e. $k = 1$) the expectation of the initial guess $\mathbf{X}^{(0)}$ is given by the current belief in the parameters: $E (X^{(0)}) = E (\theta_{0,c})$ and the variance by the average of the previously EM–estimated variances: $V (X^{(0)}) = \frac{1}{l} \sum_{l=0}^{l-1} \mathbf{C}^{(K_{l})}$ (where $\mathbf{C}^{(K_{l})} = \mathbf{C}_{0}$, the variance of the initial guess, $\mathbf{X}_{0}$).

The precision of the calculations will depend on the number of samples in the simulations, $Z^{(k)}$, and it may be beneficial to raise this number with the iteration number, $k$, so that the precision will increase as the parameter values converge.

The EM1 algorithm is summarized as follows:

- Collect all data on all pigs in all batches terminated at the current updating step. No calculations/updates will be performed if no batches are terminated.
- For each logical group estimate the “unbiased observed mean” by applying the EM algorithm.
- For each logical group update the belief in expectation by applying the Kalman filter.
3.2 Models and methods

Some basic results from applying the method to a simulated data set are shown in fig. 3.3. The simulation settings and the data set used are described in details below in the model testing and results sections, but it should be noted that batches contain an average of approximately 95.8 pigs and that pigs are selected (as demonstrated in table 3.1 above) at a live weight of 93 kg and over 5 weeks (/stages as illustrated in fig. (3.1)). Monte Carlo simulation has been applied in order to generate approximately 200 samples per pig per iteration. Results (live weight at stage 3, $W_3$) of 99 iterations are shown for the first batch given 5 different levels of information: (i) Live weight ($W$) and carcass weight ($CW$) observed only once (i.e. only 20% of the variables are observed on each individual) – at slaughter – and perfect initial guess (see the models and methods testing section below and Appendix 3.A); (ii) As (i), but in addition $W$ also observed at stage 1 (i.e. in average less than 30% of the variables observed); (iii) As (i) but in addition $W$ observed at all stages prior to marketing (i.e. approximately 45% of the variables observed); (iv) As (iii), but a poor initial guess, $X_0$ (expectations, correlations and $V$ ($W$) have been reduced by 20% while $V$ ($CW$) has been increased by 20%); (v) “perfect information” (see the model testing section below), which might be used as a reference point. The observations are due to selection highly biased and as shown in the fig. 3.3 the value of $W_3$ converges towards a value close to the true mean (case (v)) only in the situations of high levels of information (case (iii) and (iv)). It should be noted that this very rapid and precise convergence does not apply to all variables; e.g. the live weight at stage 5 converges to a stationary value (which differs from the true value by $-0.49$ kg) after more than 80 iterations.

In the cases of small levels of information (case (i) and (ii)) the stationary values of the estimates (after 99 iterations) are highly biased and far from the mean given perfect information. Two different approaches based on different assumptions have been applied in order to reduce this bias:

- Selection criterion assumed known.
- Shape of growth function assumed known.

These two approaches are presented below.

\[ W_3, \text{ live weight at stage 3} \]

\[ \text{Weight (kg)} \]

\[ \text{Iteration \#} \]

\[ \text{W and CW observed once} \quad \text{W observed twice} \quad \text{All W observed} \]

\[ \text{All W observed, bad prior} \quad \text{Observed mean, perf. info.} \]

**Figure 3.3.** Convergence of the EM algorithm, given different levels of information. Results for the 1st batch (96 pigs).
3.2.4.3 Selection criteria known – the EM2 algorithm.

As discussed in the introduction, methods exist for the special cases in which data are missing because of grouping/censoring/selection of individuals/data. In the following it is assumed that pigs are selected based on live weight (i.e. they are selected if the live weight exceeds a predefined threshold weight) and an extension of the EM1 algorithm as defined above is introduced in order to utilize this assumption which can be regarded as additional information.

Assuming that the selection criterion is based solely on observable traits which are represented in the model, the criterion can be modeled by a set of truth valued (i.e. Boolean) functions with the traits, $x$, as argument:

$$v_n(x) = \begin{cases} 
-\zeta_n(x) \land v_{n-1}(x), & n \geq 1 \\
\text{TRUE,} & n = 0 
\end{cases}$$

$$\tau_n(x) = \zeta_n(x) \land v_{n-1}(x)$$

where $v_n$ is a truth valued function returning TRUE if a pig with observed traits $x$ has not been selected at stage $n$ or a previous stage and otherwise FALSE,

$\zeta_n$ is a truth valued function returning TRUE if a pig with observed traits $x$ satisfies the selection criterion at stage $n$ and otherwise FALSE and

$\tau_n$ is a truth valued function returning TRUE if a pig with observed traits $x$ is selected at stage $n$ and it has not previously been selected and otherwise FALSE.

In the case of selection based on observed live weight, $W_n$, the function $\zeta_n$ is defined by:

$$\zeta_n(x) = \begin{cases} 
(w_n > w^*), & n = 1, \ldots, N-1 \\
\text{TRUE,} & n = N 
\end{cases}$$

where $w_n$ is the observed live weight (i.e. an element of the complete observable vector $x$) and

$w^*$ is a given threshold weight (same at all stages and explicitly given).

The distribution in (3.7) in the E–step of the EM1 algorithm is conditioned on the selection criterion and it is redefined as:

$$\langle x_{iz} \rangle \sim f_{X_{iz} \mid \tau_n(x)}, \quad z = 1, \ldots, Z^{(k)}$$

(3.9)

where $n$ is the actual marketing stage of the individual, $i$. 
In contrary to the samples in (3.7), all samples in (3.9) satisfy the selection criterion and the ML estimates given by (3.8) are consequently conditioned on the actual observations as well as the selection criterion. It should be noted that in some rare cases (i.e. when the initial guess on the distribution is very divergent from the true underlying distribution) it might, from a numerical point of view, be difficult or even impossible to sample from the distribution given by (3.9). In those cases (3.7) is applied in the iterations until sampling from the conditional distribution (3.9) is possible.

Basic results of this revised EM algorithm, the EM2 algorithm, applied to the same data set and conditions (i.e. initial guesses) as in fig. 3.3 are shown in fig. 3.4. The conditional sampling (with perfect initial guess) reduces the error (i.e. the difference between the true observed mean with perfect information and the estimated mean) on the estimates in the figure from −9.61 to 1.38 kg and the errors on the other 9 variables are even lower (0.46 kg in average). In the situation of a “poor initial guess” (as defined in the EM1 algorithm above) the convergence of the algorithm is worse – the convergence is slower and the stationary values on the other variables than $W_5$ are more divergent from the true values. The performance of the algorithm will be examined in more details in the results section below.

3.2.4.4 Shape of growth function known – the EM3 algorithm

Studies of the growth of pigs (see the introduction for references) have shown that growth of pigs generally follow commonly shaped growth curves and that the relation between traits (e.g. live weights and carcass weights) and over time (e.g. live weight from week to week), because of the underlying biological homogeneity, does not differ much between comparable, modern slaughter pig finishing operations. This implies that prior to observing any of the pigs in a given operation the manager will have a very good (i.e. close to the true) belief in the traits of the pigs or at least in the “shape (but not the level)” of the mean growth curves” and the relations between traits.

In the examples in fig. 3.3 and fig. 3.4 it was demonstrated how the parameter estimates of the EM algorithm converges to stationary values far from the true values when the informational level is low (i.e. each individual observed only once and no information of the selection criterion). However, it was also demonstrated that in the cases of a “perfect initial guess on the distribution” these errors after a single iteration were of very little magnitudes; with a good initial guess on

![Figure 3.4. Convergence of the EM algorithm, with and without the assumption that the selection criterion is known and given different initial guesses. Results of the 1st batch (96 pigs).](image-url)
“shape of the mean growth curve” (i.e. relation between the expectations of the initial guess) and the “relations between the variables” (i.e. variances and correlations of the initial guess), a single iteration appears to return estimates of the expectations that even in the case of a very poor informational level are close to the stationary values with “perfect information”.

These two features (the common knowledge of growth of pigs and the behavior of the EM algorithm with a “good initial guess”) can be utilized in order to simplify the calculations, to relax the assumption of known selection criterion and to estimate parameter values in cases of very little informational levels (i.e. pigs only observed at slaughter). The revised EM algorithm is as follows.

Let $y_t$ and $V_t$ in the Kalman filter (3.4) be given by:

$$y_{lt} = S_l / n_{lt} \quad (a)$$

$$V_{lt} = \left( (M_l - S_l S_l^T / n_{lt}) / n_{lt} \right) + \frac{1}{n_{lt}} \sum_{i=1}^{n_b} C_i \quad (b)$$

where:

$$E(X^*) = m_0, \quad V(X^*) = C_0,$$

$$m_i^* = E(X^* | y_{lt}, \varepsilon_i) \quad and \quad C_i^* = V(X^* | y_{lt}, \varepsilon_i), \quad i = 1, \ldots, n_{lt}$$

and:

$$M_l = \sum_{i=1}^{n_b} m_i^* \cdot m_i^T \quad and \quad S_l = \sum_{i=1}^{n_b} m_i^*$$

It should be noted that the initial guess, $X_0$, is used in all updating steps.

The observational variance $V_t$ depends on the variance on the mean of the conditional expectations of the individuals, $m_i^*$, as well as on the conditional variances on individuals, $C_i^*$. This dependence on the conditional variances is in the basic EM algorithm achieved by the sampling in (3.7) – a sampling that might be avoid by extending (3.8) with the second term in (3.10.b).

From (3.10) it follows that only in the situation of small conditional variance on the individuals (i.e. $C_i^*$ “little”) and a large sample size ($n_b$) the observational variance, $V_t$, will be small and the filter (3.4) will “completely” adjust to the observation, $y_t$. It should be noted that even with a “perfect initial guess” the estimates will be severely biased if the relations between traits are low – i.e. correlations are close to 0. However, as will later be demonstrated, the correlations in the animal growth models applied/estimated here are in most situation high, above 0.9.

This method will be referred to as the EM3 algorithm.

3.2.5 Models and methods testing
The models and methods are tested on simulated data using the model developed and described by Jørgensen (1993) and Jørgensen & Kristensen (1995). Simulated data have been chosen for
the testing in order to get full information of all traits at all marketing stages; In the simulation model it is possible to generate data on all traits (even latent traits) of all pigs at all stages and the “true distribution” of traits or the “true animal growth model” can be estimated directly based on data and used as a reference in the calculations.

The simulated production system is partitioned into 8 marketing sections, each containing 100 pigs. Pigs remain in the productions system for a maximum of 16 weeks (including 1 week for cleaning etc.). Every 2nd week a new batch is initiated. Pigs are marketed over no more than 5 weeks with 1 marketing/week and in average 50 pigs are marketed every week. Only live weight $[W_{i1}, \ldots, W_{i5}]$ and carcass weight $[CW_{i1}, \ldots, CW_{i5}]$ have been simulated and measured implying that the state of a pig’s traits at all 5 marketing stages is represented by:

$$X_i = [W_{i1}, CW_{i1}, W_{i2}, \ldots, W_{i5}, CW_{i5}]^T$$

The state of the herd is represented by a single logical group.

A sample data set was generated by simulating 800 days of production. The simulation model was calibrated according to parameter estimates based on a data set from The Danish Slaughter Houses (1011 samples from an experiment first published by Udesen, 1993) in which only 5–10% of the pigs have been selected for marketing and on a comprehensive (i.e. many longitudinal observations) data set (189 samples) from a currently unpublished experiment at the Royal Agricultural and Veterinary University, Copenhagen (Staun, 1996). All traits of all (surviving) pigs were generated and observed at all stages (i.e. a “perfect information” data set) in the simulations, and data sets with lower levels of information were generated by extracting sub–sets of this “perfect information” data set; all data sets were sub–sets of the output of a single simulation run. Model parameters of the “perfect initial guess” were estimated based on the “perfect information” data set. The value of the parameters are shown in Appendix 3.A. In order to test the sensitivity of the models concerning the initial guess a second initial guess with poor (i.e. very divergent from true) parameter values was applied. The parameters of this “poor initial guess” are shown in Appendix 3.A.II.

Beliefs were updated weekly (i.e. a 1 week interval between update steps, $t$) and parameter values and basic statistics were calculated and recorded. Major model parameters were changed and models (bias elimination and belief updating) reran to test the sensitivity of the methods with respect to errors in model parameters and assumptions. The type and magnitude of these changes will be described and discussed in the Results section.

In order to increase sample size and improve (co)variance estimates the EM1 and EM2 algorithms were applied to all (2309) pigs of the first 24 marketings directly without intermediate Kalman filtering (i.e. no dynamic updating).

To test the methods’ ability to capture and adjust to changes in the state of the herd a second data set has been generated. The growth rate of pigs was in this simulation run increased after 150 days, implying a significant increase in expectations (e.g. a change in $E(CW_{5i})$ from 81.8 to 87.60 kg) and a slight decrease in variation. As the growth rate was changed for all pigs in the system at the time of the change, the effect of the change would not be fully effective until $150 + 77 = 227$ days (32 weeks) after the change.

The variance discount factor ($\delta$) was set to 0.9 (i.e. a little adaption to observations – no changes in the production system are expected in the default situation) and the initial prior of the Kalman filter was set to 80% of the initial guess of the animal growth model; $M_0 = 0.8 \cdot C_0$. 

3.2 Models and methods
3.3 Results

Parts of the results (live weight at stage 1 (i.e. at the first marketing), $W_i$ and carcass weight at stage 3, $CW_i$) from the “truncated mean transformation method” (in the following denoted as the TMT method) with perfect information (i.e. all traits of all pigs observed at all marketing stages) and after Kalman filtering are shown graphically in figure 3.5. Observations are in general simple sample means of in average 96 surviving pigs, but because of the design of the experiment (insertion of weaners every second week) there will every second week be no observations on a given trait and observations as shown in the figure are in those situations based on observations on other traits (i.e. the conditional (3.2)). This explains the tendency of pairwise observations on particularly $W_i$; first observation is the true observation, while second observation is conditioned on observations on the same animals a week later.

The updated beliefs in expectations of the state of the pigs are underestimated in the beginning, but they are stabilized around the true expectation after approximately 30 updates (or marketings). The filter is set to be a little more adaptive in the beginning (i.e. a vague initial prior belief) than at succeeding stages and consequently it adjusts more thoroughly to the first observations than succeeding observations. However, in the experiment the first observations happens to be below the true expectation and it takes time (or updates) for the filter to adjust for this initial flaw. The 95% confidence intervals are based on the belief in variances (i.e. $M_y$ in (3.4)) and represents the confidence in the estimates of expectations given the observations and the observational (i.e. $v_y$ in (3.4)) and system errors (i.e. $w_y$ in (3.4)).

The results based on perfect information can be used as a reference or a upper common limit on model performance when interpreting results based on imperfect information; no results should be expected to be better than the results based on perfect information. In order to compare the different methods, the mean of the absolute mean errors (MAME) is in all cases calculated.

![Figure 3.5. Model output: Truncated mean transformation (TMT) with perfect initial prior and perfect information (i.e. all traits of all pigs observed at all stages). “Unbiased” observations ($y_{W1}$, $y_{CW3}$), true expectations ($m_{W1}$, $m_{CW3}$), estimated expectations ($m^{^\wedge}_{W1}$, $m^{^\wedge}_{CW3}$) and 95% confidence intervals on estimates.](image-url)
3.3 Results

![Graph](image)

Figure 3.6. Model output: TMT, with perfect initial prior belief. “Unbiased” observations (y\_W1, y\_CW3), true expectations (m\_W1, m\_CW3), estimated expectations (m^\_W1, m^\_CW3) and 95% confidence intervals on estimates.

as follows: For each of the 10 variables calculate the mean of errors (ME) from the time of stabilization of the filter (mean of differences between estimated and true expectations from update # 18 to update # 102 – only the first 74 updates are shown in the figure); MAME is then given as the mean of absolute ME of the 10 variables. The MAME on the results presented in fig. 3.5 is 0.02 kg.

In figure 3.6 the results of the TMT method based on incomplete (and biased) observations are shown. The “unbiased observations” are more scattered than in the case of perfect information and the beliefs in expectations are in general (and systematically) overestimated (approximately 0.45 kg for \(W_1\) and 0.24 kg for \(CW_3\), see table 3.3 below). The 95% confidence intervals have been extended due to reduced sample sizes; pigs are only observed once in their lifetime and consequently the observational errors (i.e. the variances on sample means) are increased. MAME is 0.34 kg.

Results of the basic EM algorithm, EM1, with live weight observed prior to and at the time of marketing and carcass weight observed at marketing, are shown in fig. 3.7. Pigs are continuously observed (i.e. when marketed), but as observations are accumulated until the last marketing from a batch (which happens every second week in the simulated slaughter pig operation) the number of updates have been reduced to approximately the half compared to applying the TMT method on the same data set. In average approximately 50 samples per pig was generated in the Monte Carlo simulations in the E–step of the algorithm and convergence (\(\epsilon = 0.05\) kg) was (as demonstrated in fig. 3.3) reached after 80 iterations at step 1 and less than 10 at succeeding steps. The “poor initial guess” used in the example is described in the models and methods section above. As demonstrated by the example the method quickly corrects this bad initial guess and the MAME (based on update #8 to 50 – only the first 42 updates are shown in the figure) is as little
as 0.06 kg. The estimated covariance matrix and correlation matrix after 50 updates are shown in Appendix 3.B.1 as divergences from the “true” values in appendix 3.A. Despite the very simple updating method for variances (i.e. simply the mean of variances over all updating steps) the estimates are (with respect to the overall objective of the model as a preprocessor for a marketing management DSS system) close to the true values, with a tendency to overestimate variances, but with good estimates of correlations. As being demonstrated below, other updating methods (e.g. ML estimates based on all pigs marketed prior to a given updating step) may result in even better estimates. It should be noted that in the case of no longitudinal data (i.e. pigs only observed and registered at marketing) the MAME is 6.18 kg, with a max. ME on \( W_1 \) (12.75 kg).

The second EM algorithm which is based on the assumption that the selection criterion is known (EM2) has been applied with the same “bad initial guess” as in EM1, but with a lower level of information; live weight as well as carcass weight are observed at the time of marketing only. The MAME is 0.42 kg with a maximum absolute ME on \( W_5 \) (−0.90 kg) and as is the case with EM1 acceptable estimates of covariances have been achieved after 50 updates (see appendix 3.B).
3.3 Results

![Graph showing model output for EM3 with perfect initial prior belief](image)

**Figure 3.8.** Model output: EM3 with “perfect initial prior belief”. “Unbiased” observations (Y_{W1}, Y_{CW3}), true expectations (m_{W1}, m_{CW3}), estimated means (m^{\wedge}_{W1}, m^{\wedge}_{CW3}) and 95% confidence intervals on estimates.

Results of the third EM algorithm, EM3, with perfect initial guess and two observations per individual (live weight and carcass weight at slaughter) are shown in fig. 3.8. Because of the perfect initial prior the error on estimates is little at the first updates and the MAME is 0.11 (see also row #2 in table 3.3).
3.3.1 Sensitivity of TMT and EM3

The TMT and the EM3 methods are both based on assumptions concerning those model parameters not estimated/updated by the methods. In table 3.3 the results of the two methods given partial changes/errors in major model parameters (initial guess on distributions and animal growth models) are summarized. Mean errors and standard deviation on errors as well as MAME are shown.

The two basic situations presented above (i.e. perfect information and perfect initial guess) are represented by row #1 and 2 respectively. Changes/errors in variances (row #3–5) does not affect the results of EM3, while the TMT method is more sensitive; a 30% decrease in variances results in a mean error on \( W_1 \) (i.e. the live weight at marketing stage 1) of 0.93 kg. Changes in correlations (row #6 and 7) and changes in variances as well as in correlations (row #8) have almost the same effect; the EM3 method appears to be very robust, while the TMT method is quite sensitive.

In row #9 and 10 the parameters (mainly the expectations) have been changed indirectly by changing the growth rate in the underlying simulation model (from which samples are generated and model parameters estimated). In neither cases the ME or MAME on the two methods have

<table>
<thead>
<tr>
<th>Model parameter(s) changed</th>
<th>Mean Error (std), kg</th>
<th>TMT</th>
<th>EM3</th>
<th>TMT</th>
<th>EM3</th>
<th>MAME</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1 Perfect information(^1)</td>
<td>.03 (1.16)</td>
<td>.02 (1.18)</td>
<td>.02 (1.10)</td>
<td>.02</td>
<td>.02</td>
<td></td>
</tr>
<tr>
<td>2 No change</td>
<td>.45 (1.16)</td>
<td>.24 (1.16)</td>
<td>.11 (0.09)</td>
<td>.34</td>
<td>.11</td>
<td></td>
</tr>
<tr>
<td>3 V (W), V (CW): +30%</td>
<td>-.00 (2.8)</td>
<td>.19 (2.0)</td>
<td>.11 (0.09)</td>
<td>.34</td>
<td>.11</td>
<td></td>
</tr>
<tr>
<td>4 V (W), V (CW): -30%</td>
<td>.93 (2.6)</td>
<td>.27 (1.12)</td>
<td>.11 (0.09)</td>
<td>.96</td>
<td>.11</td>
<td></td>
</tr>
<tr>
<td>5 ( \rho (\cdot, \cdot): -10% )</td>
<td>-.16 (1.10)</td>
<td>.23 (1.12)</td>
<td>.12 (0.08)</td>
<td>.53</td>
<td>.10</td>
<td></td>
</tr>
<tr>
<td>6 ( \rho (W_1, W_2): -35%, \rho (\cdot, \cdot): -20% )</td>
<td>1.37 (0.69)</td>
<td>.16 (0.17)</td>
<td>.12 (0.08)</td>
<td>1.21</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>7 as row #7, but V (W): +20%, V (CW): -20%</td>
<td>2.49 (0.28)</td>
<td>.58 (0.10)</td>
<td>.13 (0.08)</td>
<td>2.00</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>8 Poor initial guess</td>
<td>1.08 (0.11)</td>
<td>.52 (0.16)</td>
<td>-.12 (0.12)</td>
<td>.93</td>
<td>.28</td>
<td></td>
</tr>
<tr>
<td>9 ( k_1: -10%(^2)</td>
<td>.24 (0.13)</td>
<td>.19 (0.14)</td>
<td>-.04 (0.08)</td>
<td>.19</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>10 ( k_1: +15%(^3)</td>
<td>.48 (0.16)</td>
<td>.27 (0.17)</td>
<td>.22 (0.09)</td>
<td>.36</td>
<td>0.24</td>
<td></td>
</tr>
<tr>
<td>11 No change, W not observed</td>
<td>.37 (0.16)</td>
<td>.24 (0.16)</td>
<td>.11 (0.09)</td>
<td>.32</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>12 ( k_1: -10%, W not observed(^2)</td>
<td>.18 (0.14)</td>
<td>.19 (0.14)</td>
<td>-.19 (0.06)</td>
<td>.17</td>
<td>0.25</td>
<td></td>
</tr>
</tbody>
</table>

\(^1\) All variables of all (living) pigs observed at all stages.

\(^2\) \( k_1 \) is a parameter in the simulation model. Parameters of initial guess distributions and animal growth models are indirectly changed (e.g. \( E(W_1) \) is reduced by 7.4% and \( E(CW_2) \) by 7.6% when \( k_1 \) is reduced by 10%).
been increased (or decreased) significantly; the absolute levels of the expectations of the initial guesses are of less importance as long as the relations between traits (and expectations) and the “shapes of mean growth curves” are maintained.

In the final 2 rows (# 11 and 12) only the carcass weights have been observed. This is the normal situation in most slaughter pig operations and as shown in table 3.3 the ME on both methods is only slightly changed compared to the situations of observations on carcass weight as well as live weight (i.e. row # 2 and 9 respectively). In the case of errors in expectations (i.e. initial guess for TNT and the animal growth model for EM3) on either of the traits, the gain (i.e. the reduction in ME) of observing both traits may be increased. However, because of a very close relationship (which can be estimated based on a few samples) between carcass weight and live weight the gain of this additional information (on live weight) will in most situations be of very little magnitude.

3.3.2 Adaption to changes in growth rate

To illustrate and test the abilities of the Kalman filter (or the DLM) the EM3 method have been applied to the second data set in which an intermediate system change have been simulated (i.e. a 10% increase in growth rate after 150 days). The results are shown in figure 3.9. The expectation of the initial prior is underestimated by 10%, but a high initial adaption rate (i.e. a high initial variance on (or uncertainty in) the belief, $M_{10}$ in (3.4)) results in a quick adjustment of the belief in expectation towards the true expectation. It should be noted that the “slow” growth model (in which e.g. $E(W_i)$ is 86% of $E(W_i)$ after the system change) presented above (i.e. row # 9 in table 3.3) has been used in the EM3 algorithm.

The manager of the slaughter pig operation is assumed to know that the state of the system is going to change (the change might even be induced by the manager), but the expected magnitude of the change is unknown. In the filtering this knowledge is modeled as an instant increase in uncertainty (i.e. in $M_{10}$) at the time of the expected full effect of the system change, resulting in an instant increase in the adaption rate as illustrated in figure 3.10. Consequently the filter adjusts.
very rapidly to the first new observations (i.e. at update # 8) and after a few updates the filter has re–stabilized and the variances (confidence intervals in figure 3.10) on estimates have decreased to the same level as before the system change.

3.3.3 Covariance estimation

The results of estimating model parameters (and in particular (co)variances) based on the first 24 batches (2309 pigs) are shown (as divergences from the situation of perfect information, Appendix 3.B.III) in Appendix 3.B.IV and 3.B.V. Only in the case of EM2 the increased sample size (i.e. from approximately 96 to 2309 individuals) results in better estimates compared to the basic Kalman filter estimates presented above and in Appendix 3.B.II. In particular the large errors on variances has been reduced (e.g. from 13.40 kg² to 4.63 kg² on CW1), but not eliminated. Applying additional information/assumptions (like e.g. the assumption of a constant increase in variances over time, see also the Discussion below) may reveal even better results.

3.4 Discussion

In the Models and methods and Results sections it was demonstrated how different levels of information can be handled and parameters estimated by applying different assumptions and methods. In the case of a high level of information (live weight observed at all stages prior to marketing) the “belief in the herd” can be estimated with high accuracy directly based on data, while in the case of the lowest level of information (aggregate data, no identification between farm and packer), assumptions on (co)variances as well as selection criterion were applied in order to get satisfactory estimates. The sensitivity of the methods with respect to errors in the specific assumptions has been analyzed in the results section, but errors in the general assumptions concerning identification of individuals and normally distributed variables have not been analyzed nor discussed.
As discussed in the introduction pigs are normally not individually identified; pigs are simply members of a group (e.g. a batch), but even this group identification might be blurred or even missing, e.g. during marketing. The methods presented here rely heavily on the assumption that the group membership of each individual can be identified and missing group identification may be the biggest problem in applying/implementing the proposed methods in practical slaughter pig marketing management. Another (minor) problem may be imposed by non–normal distributions of traits. In general managers may treat different pigs differently (e.g. “poor” performing pigs are “helped” to catch up with better performing section–mates) in order to increase profitability. As a consequence of this discrimination (and selection) the expected natural normal distribution of traits are disturbed. However, based on an analysis of the empiric data material presented in the Models and methods section above, the assumption of normally distributed traits cannot in general be rejected and most errors resulting from non–normal distributions can be eliminated by choosing appropriate logical groups.

All three EM based methods are quite insensible to errors in assumptions/initial guesses and in most cases return acceptable estimates. The actual choice of a method (TMT, EM1, EM2, EM3) in a particular situation depends mainly on the informational level, but the knowledge of/confidence in a general or herd specific animal growth model might influence the choice between EM2 and EM3. TMT and EM3 are from a computational point of view the most appealing methods as the simulations and iterations of the basic EM algorithm (which can be very processor consuming – hours of processing time on a Pentium based Personal Computer) are avoided, but the two methods do not return estimates of (co)variances.

The output of the methods (i.e. parameter estimates) is primarily intended for input in a DSS (see the introduction), but as it represents the current (updated) belief in the herd it may as well serve other management purposes, including a general monitoring of animal performance.

The proposed animal growth model was formulated as a strictly empiric model specifically for the purpose of slaughter pig marketing management support, but it may very well prove useful as a stochastic extension of traditional deterministic, mechanistic and continuous growth models (see the introduction). In the simplest case, predictions of expectations of the proposed animal growth model can be assessed by applying an empiric and deterministic growth model in a more traditional fashion (i.e. given some changes in the system, e.g. feeding). In the more general situation the special structure of variances and correlations in the animal growth model and as exemplified in fig. 3.11 can be utilized; Variances and correlations of the model can be modeled by functional relations (as indicated by the lines connecting the points in the figure). Variances are then given as a function of time (stage in the discrete animal growth model (3.1)) and correlations as a function of two times (two stages). An example to illustrate this: Using the functional relationships in fig. 3.14 the correlation between live weight at stage 1 and carcass weight at stage 2.5 would be approximately 0.936 (indicated by a “□” in the figure).
The Kalman filter proved to be useful in dynamically updating parameter estimates. In the situations of a steady production (i.e. no change in growth rates) more simple methods like a moving average or the EM algorithm applied to all pigs of the last 5 or 10 marketings may return the same or even better results, but in the case of a system change the true power of the filter was revealed: The ability to quickly adjust to sudden changes and the option of the model builder/user to intervene and supply additional information. The simplest filter (the steady DLM) was chosen for the purpose, but other more sophisticated models, including multi level models and seasonal models (see e.g. Harrison & West, 1989) can be applied in order to improve model performance and longer term predictions (e.g. from month to month or quarter to quarter).

Simulated data were used for model testing primarily in order to get full information on all traits – information that due to the latent traits does not exist in the “real world”. The performance of the methods was evaluated with respect to the (known) “world of the simulation model” and should not be interpreted as the “true” performance of the methods without precautions; final conclusions should be based on empiric data and testings/validations in “the real world”. However, assuming that the applied simulation model is approximately representing “the real world” the proposed models and methods can form the basis of a slaughter pig marketing belief management system, which might serve as a direct preprocessor of the DSS presented in chapter 4 (with the animal growth model as an interface), as a more general monitoring and management support system and as a general method for estimating model parameters based on highly biased data.

Figure 3.11. Correlations between W and CW. Data from the “perfect initial guess” of appendix 3.A.I.
3.5 References


3.4 Parameters of the initial guesses

Appendix 3.A. Parameters of the initial guesses on distributions

3.A.1 “Perfect initial guess”

ML estimates based on the first 4700 simulated pigs (approximately the same number of pigs as in the first 49 Kalman filter updates) with all traits observed at all stages (i.e. with “perfect information”). Expectations (in first column), variances (in diagonal), covariances (above diagonal) and correlations (below diagonal and in italic font):

\[
\begin{bmatrix}
81.72 & 56.86 & 44.72 & 58.86 & 46.49 & 60.63 & 47.99 & 62.33 & 48.88 & 63.07 & 49.91 \\
64.55 & .957 & 38.41 & 46.64 & 36.83 & 48.04 & 38.01 & 49.37 & 38.81 & 49.96 & 39.56 \\
87.35 & .979 & .943 & 63.63 & 49.93 & 65.18 & 51.55 & 66.97 & 52.59 & 67.82 & 53.60 \\
69.01 & .943 & .909 & .957 & 42.77 & 51.40 & 40.67 & 52.91 & 41.58 & 53.64 & 42.45 \\
92.90 & .965 & .931 & .981 & .944 & 69.37 & 54.63 & 70.96 & 55.65 & 71.91 & 56.80 \\
73.39 & .929 & .895 & .943 & .908 & .958 & 46.92 & 56.18 & 44.10 & 56.90 & 44.96 \\
98.31 & .954 & .919 & .969 & .934 & .983 & .946 & 75.10 & 58.71 & 75.76 & 59.82 \\
77.63 & .915 & .883 & .930 & .897 & .943 & .908 & .956 & 50.24 & 59.52 & 47.12 \\
103.60 & .941 & .907 & .957 & .923 & .972 & .935 & .984 & .945 & 78.95 & 62.05 \\
81.86 & .905 & .872 & .918 & .887 & .932 & .897 & .943 & .909 & .954 & 53.54
\end{bmatrix}
\]

3.A.2 “Poor initial guess”

Expectations and correlations reduced by approximately 10% and variances by approximately 20% compared to the parameters of the “perfect initial guess on distribution” above. Expectations (in first column), variances (in diagonal), covariances (above diagonal) and correlations (below diagonal and in italic font):

\[
\begin{bmatrix}
73.54 & .4532 & .3127 & .4234 & .3246 & .4365 & .3352 & .4482 & .3430 & .4541 & .3486 \\
58.10 & .874 & .2826 & .3248 & .2641 & .3348 & .2727 & .3436 & .2789 & .3482 & .2836 \\
78.61 & .884 & .858 & .5067 & .3495 & .4689 & .3599 & .4812 & .3685 & .4879 & .3747 \\
62.10 & .858 & .884 & .874 & .3156 & .3594 & .2926 & .3689 & .2996 & .3741 & .3047 \\
83.61 & .872 & .847 & .886 & .860 & .5532 & .3822 & .5102 & .3906 & .5177 & .3974 \\
66.05 & .847 & .872 & .860 & .886 & .874 & .3457 & .3917 & .3181 & .3974 & .3236 \\
88.48 & .861 & .836 & .874 & .849 & .887 & .861 & .5981 & .4120 & .5451 & .4185 \\
69.89 & .836 & .861 & .849 & .875 & .861 & .887 & .874 & .3716 & .4173 & .3398 \\
73.68 & .825 & .850 & .839 & .864 & .851 & .877 & .862 & .888 & .874 & .3938
\end{bmatrix}
\]
Appendix 3.B. Estimated parameter values

3.B.I  Estimated values for EM1

Estimates after 50 Kalman filter updates (with EM iterations in each update), with the “poor initial guess” and with all live weights observed prior to marketing and live weight and carcass weight observed at marketing (see also fig. 3.7 in the results section). Deviations from the “true” values in 3.A.I:

\[
\begin{bmatrix}
0.03 & -0.85 & 0.13 & -0.42 & 0.48 & -0.38 & 2.42 & 0.04 & 1.26 & -0.01 & 0.87 \\
-0.17 & -0.01 & 0.86 & 0.89 & 2.05 & 1.45 & 3.82 & 2.12 & 3.18 & 2.24 & 2.76 \\
0.00 & 0.04 & 0.11 & -0.47 & 0.62 & -0.18 & 2.80 & 0.31 & 1.63 & 0.24 & 1.17 \\
-0.11 & 0.00 & 0.23 & -0.02 & 1.59 & 1.08 & 3.92 & 1.76 & 3.08 & 1.76 & 2.29 \\
0.04 & 0.04 & 0.20 & 0.03 & 0.05 & -0.35 & 2.97 & 0.38 & 1.70 & 0.28 & 1.11 \\
-0.12 & 0.06 & 0.32 & 0.06 & 0.22 & 0.05 & 4.95 & 3.76 & 4.80 & 4.14 & 4.52 \\
0.04 & 0.03 & 0.24 & 0.03 & 0.09 & 0.03 & 0.09 & 0.73 & 2.36 & 0.99 & 1.61 \\
0.02 & 0.06 & 0.37 & 0.07 & 0.25 & 0.06 & 0.25 & 0.08 & 2.70 & 2.65 & 2.86 \\
0.13 & 0.02 & 0.25 & 0.02 & 0.08 & 0.01 & 0.14 & 0.03 & 0.11 & 0.84 & 1.63 \\
0.06 & 0.15 & 0.43 & 0.16 & 0.23 & 0.13 & 0.34 & 0.13 & 0.22 & 0.12 & 0.91
\end{bmatrix}
\]

3.B.II  Estimated values for EM2

Estimates after 50 Kalman filter updates, with a “poor initial guess” and live weight and carcass weight observed at marketing. Deviations from the “true” values in 3.A.I:

\[
\begin{bmatrix}
-0.01 & 8.40 & 6.10 & 7.81 & 7.73 & 9.07 & 9.14 & 7.58 & 11.65 & 6.09 & 5.24 \\
0.25 & 0.06 & 4.22 & 5.83 & 6.55 & 7.44 & 7.69 & 6.74 & 10.10 & 5.37 & 5.09 \\
0.02 & 0.07 & 0.16 & 6.54 & 6.62 & 7.55 & 7.95 & 5.93 & 10.28 & 4.15 & 3.40 \\
-0.17 & 0.17 & 0.41 & 0.08 & 6.13 & 8.27 & 9.11 & 7.43 & 11.15 & 6.06 & 5.74 \\
0.07 & 0.10 & 0.30 & 0.01 & 0.02 & 8.83 & 9.35 & 7.25 & 12.41 & 5.22 & 3.89 \\
-0.08 & 0.22 & 0.46 & 0.12 & 0.49 & 0.15 & 8.36 & 8.71 & 12.97 & 6.78 & 5.45 \\
-0.25 & 0.09 & 0.37 & -0.01 & 0.26 & 0.01 & 0.24 & 5.71 & 11.15 & 3.13 & 2.01 \\
-0.14 & 0.25 & 0.55 & 0.11 & 0.48 & 0.22 & 0.04 & 13.40 & 9.04 & 8.16 & 8.16 \\
-0.46 & 0.20 & 0.45 & 0.08 & 0.36 & 0.08 & 0.27 & 0.02 & 0.20 & 0.28 & 0.18 \\
-0.50 & 0.39 & 0.73 & 0.02 & 0.22 & 0.06 & 0.16 & 0.04 & 0.07 & 0.49 & 0.01 & -1.19
\end{bmatrix}
\]
3.B. Estimated parameter values

3.B.III Estimated values for EM1, no Kalman filtering and perfect information

Estimated parameter values after 6 iterations, based on the first 24 batches (2298 pigs) with the “perfect initial guess”, “perfect information” and without intermediate Kalman filtering. Expectations (in first column), variances (in diagonal), covariances (above diagonal) and correlations (below diagonal and in italic font):

\[
\begin{bmatrix}
81.59 & 55.97 & 44.45 & 58.05 & 46.03 & 59.87 & 47.08 & 61.57 & 48.28 & 62.60 & 49.68 \\
64.49 & \cdot\cdot\cdot & 38.41 & 46.28 & 36.75 & 47.70 & 37.54 & 48.96 & 38.42 & 49.81 & 39.58 \\
87.23 & \cdot\cdot\cdot & \cdot\cdot\cdot & 49.26 & 64.11 & 50.40 & 65.89 & 51.68 & 67.05 & 53.23 \\
68.88 & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & 42.24 & 50.76 & 39.94 & 52.29 & 41.12 & 53.33 & 42.38 \\
92.81 & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & 68.12 & 53.44 & 69.95 & 54.81 & 71.22 & 56.48 \\
73.30 & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & 94.30 & 94.70 & 55.04 & 43.10 & 56.01 & 44.47 \\
98.22 & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & 73.85 & 57.86 & 75.10 & 59.48 \\
77.54 & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & 59.93 & 46.83 \\
103.54 & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & 61.96 \\
81.78 & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot \\
\end{bmatrix}
\]

3.B.IV Estimated values for EM1, no Kalman filtering

Estimated parameter values after 200 iterations, based on the first 24 batches (2309 pigs) with the “poor initial guess” and without intermediate Kalman filtering. Deviations from the values in 3.B.III:

\[
\begin{bmatrix}
0.04 & 0.06 & 0.92 & 0.28 & 1.79 & -0.25 & 0.68 & 0.35 & 4.04 & 0.85 & 2.63 \\
-0.30 & \cdot\cdot\cdot & 1.71 & 2.05 & 4.27 & 1.92 & 3.61 & 3.08 & 6.33 & 3.94 & 4.97 \\
0.05 & \cdot\cdot\cdot & \cdot\cdot\cdot & 0.51 & 1.95 & -0.02 & 0.97 & 0.65 & 4.63 & 1.19 & 2.90 \\
-0.19 & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & 3.61 & 1.94 & 4.23 & 2.92 & 6.56 & 4.02 & 5.50 \\
-0.01 & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & -0.57 & 0.51 & 0.04 & 4.21 & 0.57 & 2.45 \\
-0.06 & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & 1.70 & 2.01 & 6.68 & 2.89 & 4.56 \\
0.07 & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & 5.19 & 1.50 & 3.20 \\
0.19 & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & 7.40 & 6.15 & 7.71 \\
0.15 & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & 2.14 & 3.65 \\
0.44 & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdot\cdot\cdot & 3.58 \\
\end{bmatrix}
\]
### 3.B.V Estimated values for EM2, no Kalman filtering

Estimated parameter values after 200 iterations, based on the first 24 batches (2309 pigs) with the "poor initial guess" and without intermediate Kalman filtering. Deviations from the values in 3.B.I:

\[
\begin{bmatrix}
0.16 & -2.38 & -0.80 & -4.44 & -0.93 & -1.10 & 2.52 & -2.33 & 1.68 & 0.68 & 1.85 \\
-0.32 & -0.06 & 0.80 & -1.71 & 2.38 & 1.89 & 5.38 & 1.14 & 4.90 & 3.69 & 4.60 \\
0.53 & 0.00 & 0.09 & -6.78 & -2.71 & -3.67 & 0.48 & -4.91 & -0.41 & -1.96 & -0.28 \\
0.03 & -0.09 & 0.39 & -0.10 & 0.95 & 0.41 & 4.92 & -0.32 & 3.37 & 2.60 & 4.28 \\
0.11 & 0.00 & 0.24 & -0.05 & -0.06 & 0.46 & 3.81 & -1.49 & 3.06 & 1.70 & 2.65 \\
-0.06 & 0.09 & 0.05 & -0.02 & 0.03 & 0.01 & 6.32 & 2.78 & 7.43 & 5.58 & 6.15 \\
0.06 & 0.06 & 0.34 & 0.03 & 0.06 & -0.02 & 0.07 & -3.35 & 1.71 & -0.33 & 1.23 \\
0.21 & 0.10 & 0.05 & 0.05 & 0.21 & 0.06 & 0.046 & 0.007 & 4.63 & 4.54 & 5.24 \\
0.11 & 0.14 & 0.04 & 0.10 & 0.018 & 0.03 & 0.12 & 0.001 & 0.12 & 2.78 & 3.86 \\
0.34 & 0.24 & 0.61 & 0.019 & 0.049 & 0.010 & 0.030 & 0.011 & 0.027 & 0.011 & 3.50
\end{bmatrix}
\]
Optimal Slaughter Pig Marketing

Abstract. “The problem of optimal slaughter pig marketing management” can be partitioned into two subproblems of: (i) How to select and when to market individual pigs from batches and (ii) when to terminate (market the remainder of) the batch and insert a new batch of weaners. The two problems can be solved independently of each other and based on basic production economics and asset replacement theory. Optimization methods for solving the marketing management problem are presented and special methods are applied in order to cope with different selection criteria, with the uncertainty and variation that exists in biological systems as the slaughter pig operation and with the operational constraints that apply to the slaughter pig operation. The results show that selection on carcass leanness as well as live weight is only slightly superior to selection on live weight only and very little financial room is left for performing the on-farm leanness measuring. The selection criteria is quite unaffected by changes in model parameters (e.g. prices and growth rates), while the optimal terminal marketing stage is more affected by such changes. Results are based on simulated data and should not be interpreted without precautions.

4.1 Introduction

The problem of optimal marketing of slaughter pigs has been examined by a wide range of authors (e.g. Rasmussen, 1973; Budde, 1974; Wendt, 1979; Jolly et al., 1980; Chavas et al., 1985; Sundermeier et al., 1987; Giesen et al., 1988; Broekmans, 1992; Jørgensen, 1993; Boland et al., 1993; de Lange & Schreurs, 1995 (based on the model of Black et al., 1986)). Many different approaches have been taken, but in general most tend not to:

- clearly define the problem of marketing management and to separate and handle different aspects of the problem.
- discuss and handle the (in most situations) strong dependence between the (internal) supply
of weaners (from the farrowing operation) and the finishing operation.

– handle the stochastic variance and uncertainty of biological processes.

Based on a close examination of the feeder pig finishing operation as a system, a precise definition of the problem of slaughter pig marketing may be revealed and models and methods for solving the problem with the aim of a decision support system and with respect to the 3 aspects mentioned above developed.

4.1.1 Slaughter pig marketing defined

The feeder (or slaughter) pig finishing operation is assumed logical and operational partitioned according to a physical sectioning of the operation facilities (e.g. confinements). The group of pigs in a section is defined as a batch; all pigs in a batch must be marketed (or moved to another section) before insertion of a new batch (group of weaners) is possible, but pigs in a batch might be marketed individually. The period from the first to the last feasible marketing from a batch is partitioned into marketing stages, each stage representing the moment (e.g. a certain day of the week) at which the manager has the option (but not the obligation) to market pigs from the batch. Usually the duration of the marketing of all pigs in a batch will be no more than 4 weeks (i.e. 5 marketings). The last marketing from a batch (i.e. marketing of all remaining pigs in the section) is defined as the terminal marketing (the batch is terminated) and it is succeeded by an insertion of new pigs or weaners.

Pigs are marketed to one or more packers (slaughter houses) and the prices paid for pigs are based on a packer–specific pricing system. The pricing system will normally favor pigs within certain carcass weight and leanness intervals as shown in figure 4.1. The results (e.g. leanness’, carcass weights and financial outcomes) of a marketing are reported on an evaluation sheet.

Feeder pig finishing operations are in general partitioned into two groups:

– Batch operations, in which pigs are housed in physically separated sections mainly in order to reduce transmissions of contagious diseases between sections and from pigs to weaners.

All pigs (the batch) in a section must be marketed before insertion of a new group of weaners (a new batch) and replacement of individuals (i.e. replacing a single pig or a group of pigs by a new pig/group of pigs, without replacing the rest of the batch) is not possible at all. However, in some situations the small remaining of batch is moved to another section from where marketing has started in order to empty the section a week earlier.

![Figure 4.1. The pricing system. (a) Slaughter pig base prices paid by Danish packers (as of 1/12/96 and including a delayed payment of 0.60 DKK/kg carcass weight). (b) Price premiums paid by Danish packers (as of 1/12/96).](image-url)
Normally the number of sections is (at least at the operational and tactical planning level) fixed and it is derived from the expected/optimal duration of a batch and the planned time between batches (i.e. number of sections = duration of a batch/time between termination of batches, e.g. 14/2 = 7 sections). Sections are dimensioned according to the expected supply of weaners (see below) and the operation is run on a very tight schedule; the time of the terminal marketing of a batch is implicitly given.

- **Continuous flow operations**, in which no strict sectioning (except the separation of pigs in pens) and consequently no strict grouping of pigs in batches exists. Pigs of different ages are housed together and because of the smaller units (pens instead of sections) and because moving and mixing pigs from different pens may be a feasible option, these operations are in general more flexible and more space efficient than batch operations. Pigs are normally sorted at insertion time in order to increase the homogeneity of mainly weight and growth potential within pens. Like in the batch operations replacement of individuals in pens is not possible, but because of the flexibility of the system, regrouping and moving of individuals or groups of animals is to some extend possible.

Despite the differences between the systems there will in this presentation be no conceptual distinction between sections and pens; pens are simply defined as small sections.

After the terminal marketing of a batch, a new group of weaners (i.e. a new batch) is immediately or during some period of time (e.g. 2 weeks), inserted into the section. The new weaners are (as illustrated in fig. 4.2) supplied from either or both of two sources:

- **Internally produced weaners.** Weaners are produced within the farm enterprise (i.e. in the farrowing operation) and usually the production of weaners is managed (and optimized) independently of the finishing operation. Normally the supply or production of weaners can be changed at the strategical level (i.e. from year to year) only. The manager may buy or sell weaners, but not without costs and risks (e.g. of pour weaners or of transmission of contagious diseases). The sizes of new batches may vary.

- **Externally produced weaners.** Weaners are bought at the market/from a dealer or the weaner supply is based on a contract. The supply may be changed at the tactical level (i.e. from month to month) and in some situations even at the operational level (i.e. from day to day or week to week).

In general the supply (i.e. of batches to the finishing operation) is managed by changing either or both of the variables:

- The output of the farrowing operation.
- Trade at the weaner market.

![Diagram](image)

*Figure 4.2. The flow of weaners from the farrowing operation (via the weaner market) to the finishing operation.*
and the demand by changing:

- The size/sectioning (i.e. size of batches)/stocking density of the finishing operation.
- The finishing time for pigs (i.e. the terminal marketing stages of batches).

All four variables are mutually dependent and subject to the conditions (concerning the type of operation and the weaner supply) described above. The terminal marketing stages of batches are, except in the situation of a very flexible external weaner supply, fixed at the operational level. However, calculations of the operational and financial consequences of changing the termination time of batches can reveal valuable information on disharmonies between the farrowing and finishing operation and information for tactical and/or strategical planning of the weaner supply (i.e. the farrowing operation) and demand (i.e. size, sectioning and/or stocking density of the finishing operation).

Due to the biological variation, the pigs will grow and deposit fat at different rates. Heavy and fat pigs are (as indicated by fig. 4.1) discriminated on price and therefore it might be beneficial to select and market individual pigs from a batch prior to the termination of the batch. In most situations the manager do have this option and at least in Denmark this option is widely used. In most studies the manager is assumed to select pigs based on the observed live weight (see e.g. Chavas et al. 1985; Jørgensen, 1993), but other traits like back–fat thickness are used as selection criteria in practice and should be taken into consideration when examining slaughter pig marketing. The traits may be difficult to define and measure and in some situations they may vary randomly for the same individual over time. The observed traits (i.e. the values of the traits as observed by e.g. the manager) may differ from the true traits and these observational errors should be included in the analysis of the marketing problem as done by Jørgensen (1993) in the case of selection on live weight.

Pricing systems may differ considerably between packers\(^1\) and the same packer may have more than one pricing system. In that situation the manager of the finishing operation have the option of choosing a specific pricing system for his marketings and in some situations it may even be beneficial to choose more than one pricing system or to market pigs to several packers.

4.1.2 Optimal marketing

The slaughter pig marketing options (or slaughter pig marketing management problems) described in the previous section are summarized as follows.

The manager in general has the options (or is faced with the problems) of:

- *How to select and when to market individual pigs.*
- *How to manage the weaner supply and demand* (in particular when to terminate a batch or in other words: when to insert a new batch of weaners) and how to market (the remainder of) a batch.

In addition to these options, the manager may in the case of a continuous flow operation have the option of regrouping (i.e. splitting and/or joining groups) and moving pigs in order to increase space utilization and homogeneity among marketed pigs.

\(^1\)In Denmark all packers in general use the same standardized pricing system (i.e. the system illustrated in fig. 4.1).
4.1 Introduction

In this presentation the problem of marketing management is restricted to:

(i) How to select and when to market individual pigs and
(ii) when, irrespectively of other aspects of the managing of the weaner supply/demand, to terminate and market batches.

The problem of how to adjust the weaner supply/demand as well as the problems of regrouping pigs and of how (i.e. to which packer) to market pigs will not be examined. However, the problem of choosing one among several packers for all marketings can be solved simply by examining (i) and (ii) for each individual packer, while special methods must be applied to solve the problem of marketing to more than one packer. In particular it is a problem to handle differences in measuring and grading systems between packers.

Assuming independence between the pigs in a section, assuming the main objective of the enterprise owner to be profit maximization and assuming that replacement of individuals is impossible, problem (i) is a basic profit maximization problem (see e.g. Doll & Orazem, 1984) of allocating the two input variables, time and pricing system and the problem might be solved independently of (ii). The optimization criterion in itself is simple and straightforward, but the special nature of the slaughter pig finishing operations (and pricing systems) complicates the problem:

(a) Biological variance,
(b) latent animal traits (on which pigs are evaluated and priced and which are not revealed until after marketing, i.e. traits of the carcass),
(c) selection method and
(d) observational errors.

These special conditions contribute to stochastic processes and uncertainty in the system and special methods must be applied to handle these aspects. The solution method of the profit maximization problem is based on calculations of marginal returns and for this an animal growth model (representing the state of, and marginal change in, the traits of animals) is needed.

Finding the best option or the optimal decision/policy in (ii) is basically a replacement problem (see e.g. Rasmussen, 1976 and Jorgensen, 1993) with the batch representing the asset and with an infinite time horizon. The general methodology of replacement problems first presented by Taylor (1920, cf. Preinrich, 1940) and the Markov decision programming techniques introduced by Howard (1960) and extended to Hierarchic Markov processes by Kristensen (1988) can be applied in order to solve and study this problem. However, a set of special conditions connected to the slaughter pig marketing management problem (as well as other biological replacement problems) can extend or complicate this problem:

(e) The bias in distributions and financial outcomes caused by the selection in (i),
(f) the short term inflexible (i.e. non–adjustable) weaner supply,
(g) the type of operation and the size of batches (i.e. sections vs. pens) and
(h) The stochastic variation of the weaner supply (i.e. variation in sizes of new batches),

and as for problem (i) special methods must be applied to handle and examine these aspects of marketing management.

It should be noted that slaughter pig marketing management is part of a larger management system of the entire enterprise. Therefore and as emphasized by Chavas et al. (1985) and Jolly
et al. (1980) the marketing management cannot (at least in principle) be optimized independently of the rest of the system and often a simultaneous optimization of marketing and production (e.g. feeding) might result in better solutions than optimizing each separately. Despite these considerations the marketing and production management are (except for the weaner supply) assumed independent.

4.1.3 Objectives

The objective of the study was to develop a basic mathematical/economical optimization model for solving the slaughter pig marketing problem as defined above by (i) and (ii) and by the conditions (a) – (h).

The intended use of the model is two-fold:
- As the basis of a Slaughter pig marketing Decision Support System to be integrated in a Marketing Management Support Tool (see the General Introduction, chapter 1) as part of a larger Management Information System (see chapter 2). The system should utilize farm specific data to produce real time marketing management support for selecting and marketing individual pigs and for managing the supply and demand of weaner.
- As a general (research–) tool for optimization and analysis of slaughter pig marketing management, including the conditions (a) – (h).

The objective of this paper is to present the model and to demonstrate and discuss the two–fold intended use.

4.2 Material and methods

4.2.1 Animal growth model

In general animal growth models can be partitioned into two groups (Oltjen, 1992; Black, 1995):
- Explanatory, mechanistic models and
- predictive, empirical models.

Mechanistic growth models are detailed and in most situations deterministic models, which are based on knowledge of the underlying physiological processes and laws of physics and chemistry and on normally a large set of model parameters which are very difficult to interpret and estimate (see e.g. Black et al., 1986; Moughan et al., 1987; Moughan et al., 1995 for examples on mechanistic growth models). In general mechanistic models are not well suited for management and control.

Empirical growth models are statistical models that describe relations between variables over time, without describing underlying processes. The models are stochastic (i.e. variations on the traits as well as expectations are modeled) and as variables in most cases are easy to observe and as the number of parameters in the models is low in relation to the number of observations, parameters are more easily estimated than in the case of mechanistic models. A main and very general and well established class of empirical growth models are based on the work by Potthoff & Roy (1964): Growth Curve Models or Generalized Multi variate Analysis of Variance Models (see e.g. von Rosen, 1991 for a review).

In this paper an empiric approach is taken, but as discussed in the introduction pigs are in
most situations marketed at certain predefined moments and hence there is no need for a representation of traits as continuous variables as in the general empirical growth curve models; only the state of the traits at the time of marketing (or the time of choosing pigs for marketing) is of potential interest. Hence, in this paper traits and growth of pigs are modeled directly by a multivariate normal distribution as defined below.

Let $M$ be the number of animal traits included in the model and $N$ the last feasible marketing stage of any batch (e.g. With the option of marketing once or not at all in a week and with a 5 weeks duration of the marketing of a batch, $N$ is 5). Let the true traits (i.e. values not influenced by measuring or observational errors) of a pig at a given marketing stage (or marketing age), $n$ and at a given time, $t$ be represented by a $M$–dimensional vector, $X_{nt}$. The traits are assumed multivariate normal distributed:

$$X_{nt} \sim \mathcal{N}(m_{nt}, C_{nt}), \; n = 1, \ldots, N$$

where $m_{nt}$ is a $(M \times 1)$ vector of means and $C_{nt}$ is a $(M \times M)$ matrix of variances and covariance.

The set of variables $X_{nt}, n = 1, \ldots, N$ are merged into a single variable $X_t$, representing a pig’s traits at all $N$ marketing stages at a given time $t$:

$$X_t \sim \mathcal{N}(m_t, C_t),$$

where:

$$X_t = \begin{bmatrix} X_{1t} \\ X_{2t} \\ \vdots \\ X_{Nt} \end{bmatrix}, \quad m_t = \begin{bmatrix} m_{1t} \\ m_{2t} \\ \vdots \\ m_{Nt} \end{bmatrix}, \quad C_t = \begin{bmatrix} C_{1t} & C_{1,2t} & \cdots & C_{1,Nt} \\ C_{2,1t} & C_{2t} & \cdots & C_{2,Nt} \\ \vdots & \vdots & \ddots & \vdots \\ C_{N,1t} & C_{N,2t} & \cdots & C_{Nt} \end{bmatrix}.$$ 

**Figure 4.3.** The animal growth model. Mean growth and distribution of live weight at 5 different marketing stages. The growth rate is seen to decrease and the variance to increase over stages.
The covariance between the partial distributions \((C_{i,j}, i \neq j)\) express the correlation between traits over time. Graphically the animal model can be partially (i.e. for a single trait) represented as shown in fig. 4.3. The mean growth curve has at each marketing stage been extended with a graph representing the distribution (and not only the confidence interval) of weight at that particular stage.

Three traits are included in the model used in this presentation: Live weight \((W_{n,t})\), carcass weight \((CW_{n,t})\) and carcass leanness \((CL_{n,t}\) the fraction of meat in the carcass) implying that \(X_{n,t} = [W_{n,t}, CW_{n,t}, CL_{n,t}]^T\).

In the remainder of this presentation the time suffix \(t\) is assumed implicitly given and therefore omitted in the notation used.

The conditional distribution (or the posterior distribution) of the true traits \(X\) given a specific observation (on any subset of the traits) and given a measuring or observational error is derived as follows: Let \(X_2\) represent the observed subset of traits of \(X\), and let \(X_1\) represent the not observed traits. Then \(X\) might be (partitioned into and) represented as:

\[
X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \sim N \left( \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}, \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \right)
\]

The observational errors on the observed traits, \(X_2\) are represented by a vector, \(\varepsilon_2\), of random variables which are assumed independent of \(X_2\), mutually independent and multi variate normal distributed with zero mean: \(\varepsilon_2 \sim N(0, E_2)\).

The observed traits (e.g. the observed live weight) are then normally distributed and defined by the model:

\[
X' = \begin{bmatrix} X_1' \\ X_2' \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_2 \end{bmatrix} \sim N \left( \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}, \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} + E_2 \end{bmatrix} \right)
\]

The distribution of \(X\) given an observation \(x_2'\) of \(X_2'\) and given the observational error \(\varepsilon_2\) is then:

\[
X|X_2'=x_2', \varepsilon_2 \sim N \left( \begin{bmatrix} m_1' \\ m_2' \end{bmatrix}, \begin{bmatrix} C_{11}' & C_{12}' \\ C_{21}' & C_{22}' \end{bmatrix} \right)
\]

\((4.1)\)

where:

\[
C_E = C_{22} + \varepsilon_2 \quad (a)
\]

\[
m_i' = m_i + C_{E}C_E^{-1}(x_2' - m_2), \quad i = 1,2 \quad (b)
\]

\[
C_{ij}' = C_{ij} - C_{iE}C_{E}^{-1}C_{2j} \quad i,j = 1,2 \quad (c)
\]
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The weekly feed consumption (FC) of the pigs is only used for calculating the feeding costs and is therefore not included as a trait in the general animal model. Instead the consumption is explicitly modeled by a Gompertz growth model (see e.g. Whittemore, 1993; Jørgensen, 1993):

\[ FC(w) = k_2 \cdot k_4 \cdot (k_3 - \ln(w)) \cdot w + k_1 \cdot w^{\beta} \]

where \( w \) is the live weight,
\( k_1 \) is the energy consumption per kg metabolic weight (FU_{p}),
\( k_2 \) is the energy consumption per kg gain (FU_{p}),
\( k_3 \) is the logarithm of outgrown weight (ln (kg)) and
\( k_4 \) is a growth rate parameter of the Gompertz growth curve.

4.2.2 General assumptions of optimizations

A set of general simplifications and assumptions apply to all considerations, deductions and results presented below:

- The (sole) objective of the manager is assumed to be profit maximization;
- Pigs are assumed independent (also within pens);
- Prices and the interest rate used for discounting monetary values are assumed known and constant over time;
- Tax and health issues (including evaluations sheets deductions) will not be considered;
- Marketing costs per pig are assumed to be independent of the actual marketing management; there is no (physical nor financial) restrictions on the number of pigs per marketing.
- No regrouping of pigs between sections is possible.

4.2.3 Selection of individual pigs

In general an individual pig should be selected for marketing (and marketed) at stage \( n \) when the expected marginal (or extra) net return at stage \( n \) (\( MNR_n \)) of postponing the marketing 1 stage is less than 0 (see e.g. Doll & Orazem, 1984; Rasmussen, 1976) or in more formal terms:

\[ MNR_n(m) = r^m \cdot ERI_n(m) - EVCI_n(m) - ERI_n(0) \quad (a) \]
\[ MNR_n(1) < 0 \quad (b) \]

where \( n \) is the marketing stage, \( n = 1, \ldots, N \),
\( r \) is the discount factor \( (1+i)^{-1} \),
\( i \) is the discount rate (i.e. on a weekly basis),
\( ERI_n(m) \) is the expected return of an individual at marketing stage \( n+m \) given selection at stage \( n \)
\( ERI_n(0) \) is the expected return of an individual at marketing stage \( n \) given selection at stage \( n \) and
\( EVCI_n \) are the expected variable costs from stage \( n \) to \( n+m \).

It is important to note that (4.2) only is valid as profit optimization criteria when: (i) replacement
of individuals is economical or technical impossible and (ii) the marginal net return is not increasing over time (i.e. the marginal net return of postponing the marketing more than 1 stage does not exceed \( MNR_m \)). In this paper (i) is assumed to be satisfied. Constraint (ii) will not always be satisfied (e.g. when pigs are very young the marginal net return of keeping them 1 week more will be negative, but it is very likely to increase into a surplus before maturity) and therefore an examination of (ii) based on empiric (or at least realistic) data is needed.

4.2.3.1 General selection criteria

In general the selection criterion is modeled by a set of truth valued (i.e. Boolean) functions:

\[
\begin{align*}
\nu_n(x') &= \begin{cases} 
\neg \zeta_n(x') \land \nu_{n-1}(x'), & n \geq 1 \\
\text{TRUE}, & n = 0 
\end{cases} \\
\tau_n(x') &= \zeta_n(x') \land \nu_{n-1}(x')
\end{align*}
\]

where \( \nu_n \) is a truth valued function returning TRUE if a pig with observed traits \( x' \) has not been selected at stage \( n \) or a previous stage and otherwise FALSE,

\( \zeta_n \) is a truth valued function returning TRUE if a pig with observed traits \( x' \) satisfies the selection criterion at stage \( n \) and otherwise FALSE and

\( \tau_n \) is a truth valued function returning TRUE if a pig with observed traits \( x' \) is selected at stage \( n \) and it has not previously been selected and otherwise FALSE.

The selection/no selection probabilities are then given by:

\[
\begin{align*}
G_n &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,E}(x,e|\tau_n(x+e)) \, dx \, de = \int_{-\infty}^{\infty} f_{X'}(x' | \tau_n(x')) \, dx' \quad (a) \\
H_n &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,E}(x,e|\nu_{n-1}(x+e)) \, dx \, de = \int_{-\infty}^{\infty} f_{X'}(x' | \nu_{n-1}(x')) \, dx' \quad (b)
\end{align*}
\]

where \( f_{X,E} \) is the joint density function of \( X \) (the true traits) and \( E \) (the observational errors),

\( f_{X'} \) is the density function of \( X' \), the observed traits as defined above in the animal model,

\( G_n \) is the probability of selection at stage \( n \) (and no selection prior to stage \( n \)) and

\( H_n \) is the probability of no selection prior to stage \( n \).

The integrals in (4.3) may be calculated by numerical integration (and in certain situations they can be solved analytically) and in this presentation simulations have been used to approximate the values (the number of simulated pigs per integration – the sampling size – is denoted \( Z \)). This method is slow and imprecise, but it is easily applicable and as detailed descriptive statistics are easily extracted it has been chosen for this presentation.

The probability of no selection prior to stage \( n \) equals the probability of no selection prior to stage
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$n-1$ subtracted the probability of selection at stage $n-1$:

$$H_n = \begin{cases} 
1 & n = 1 \\
H_{n-1} - G_{n-1} & n = 2, \ldots, N 
\end{cases} \quad (4.4)$$

Applying Bayes rule, the conditional probability ($I_n$) of selection at stage $n$ given no selection prior to stage $n$ is given by:

$$I_n = p(\tau_n(x')|\nu_{n-1}(x')) = \frac{p(\tau_n(x')\cap\nu_{n-1}(x'))}{p(\nu_{n-1}(x'))} = \frac{G_n}{H_n}, \quad n = 1, \ldots, N \quad (4.5)$$

Then from (4.4) and (4.5) it follows that:

$$H_n = H_{n-1} - H_{n-1} \cdot I_{n-1} = H_{n-1} \cdot (1 - I_{n-1}) = \prod_{m=1}^{n-1} (1 - I_m), \quad n = 2, \ldots, N \quad (4.6)$$

4.2.3.2 No selection

If no pigs are selected for marketing prior to the terminal marketing the selection criterion will for any $n = 1, \ldots, N$ be:

$$\zeta_n(x') = FALSE$$

4.2.3.3 Selection based on live weight

If pigs are selected for marketing when the observed live weight at a given stage $n$ exceeds the optimal live weight of an individual as calculated below in (4.7) the selection criterion is defined as:

$$\zeta_n(x') = \begin{cases} 
(w_n' > w_n^*), & n = 1, \ldots, N-1 \\
TRUE, & n = N 
\end{cases}$$

where $w_n'$ is the observed live weight (i.e. an element of the vector $x'$) and $w_n^*$ is the optimal live weight as defined and calculated below.

The expected return of an individual pig marketed at stage $n+m$ and given a specific observed live weight ($w'$) at stage $n$ is calculated as:
\[ ERI_n(w', m) = \int f_X(x) | W_n = w', e_{w,n} | u(cw_{n+m}, cl_{n+m}) \cdot cw_{n+m} dx \]

where \( f_X \) is the density function of \( X \), conditioned on the observed live weight \( w' \) and the observation error \( e_{w,n} \) at stage \( n \) (i.e. (4.1) in the animal model above) and \( u \) is the price function (i.e. figure 4.1).

The optimal live weight (or threshold value, \( w_n^* \)) at a given stage \( n \) is defined as the lowest weight \( w' \) for which the marginal net return (MNR) of postponing the marketing 1 week is less than 0:

\[ MNR_n(w', m) = r^m \cdot ERI_n(w', m) - \sum_{l=1}^{m} [r^{l} \cdot VC(E(W_{n+l}| W_n = w'))] - ERI_n(w', 0) \quad (a) \]

\[ VC(w') = FP \cdot FC(w') \quad (4.7) \]

\[ MNR_n(w', 1) < 0 \quad (c) \]

where \( VC \) are the expected variable (feeding) costs as a function of the observed live weight \( w \),

\( FP \) is the price (DKK/FU\(_{\text{p}}\); 1 FU\(_{\text{p}}\) = MJ) of the feed diet and

\( FC \) is the feed consumption as a function of the live weight \( w \) as defined in the animal model.

If no such weight exists and the marginal return is negative for any observed live weight, \( w_n^* = 0 \) (i.e. always market). (4.7) is solved and the optimal live weight \( w_n^* \) is calculated using simulations as described below and an iterative procedure with fast convergence (the Pegasus algorithm, Barker & Tingleff, 1991).

4.2.3.4 Selection based on live weight and carcass leanness

Other selection criteria can be represented and derived in a similar manner. An example of selection based on live weight and carcass leanness is presented below in the Results section, with the expected return of an individual pig and the marginal net return given by:

\[ ERI_n(w', cl', m) = \int f_X(x) | W_n = w', CL_n = cl', e_{w,n} | e_{cl,n} | u(cw_{n+m}, cl_{n+m}) \cdot cw_{n+m} dx \]

\[ MNR_n(w', cl', m) = r^m \cdot ERI_n(w', cl', m) \quad (4.8) \]

\[ - \sum_{l=1}^{m} [r^{l} \cdot VC(E(W_{n+l}| W_n = w', CL_n = cl'))] - ERI_n(w', cl', 0) \]
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Table 4.1. The four selection criteria.

<table>
<thead>
<tr>
<th>Selection criterion (SC)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>No selection</td>
</tr>
<tr>
<td>II</td>
<td>Selection on observed live weight</td>
</tr>
<tr>
<td>III</td>
<td>Selection on observed live weight and observed carcass leanness</td>
</tr>
<tr>
<td>IV</td>
<td>Perfect information. Selection on actual MNR</td>
</tr>
</tbody>
</table>

4.2.3.5 Perfect information and summary of selection criteria

In addition to the 3 criteria described above, a fourth criterion based on perfect information on all traits of all pigs is introduced. Pigs are selected based on the actual (i.e. not the expected) marginal net return of the pig as shown in (4.9).

\[
\zeta_n(x') = \begin{cases} 
(r \cdot [u(cw_{n+1} - VC(w_n))] - u(cw_{n+1} \cdot cw_n) < 0), & n = 1, \ldots, N-1 \\
\text{TRUE}, & n = N 
\end{cases} \tag{4.9}
\]

The criterion is only applicable if (as in this presentation) the true value (i.e. without observational errors) of traits used for calculating the net return at current and succeeding marketing stage are known at the time of selection. Values calculated based on this (unrealistic) perfect information selection criterion represent the best or highest achievable results and might be valuable in assessing the efficiency and expected utility of other criteria. The four selection criteria are summarized in table 4.1.

4.2.4 Terminal marketing. Constant and inflexible weaner supply

The conditional probability of selection of a single pig at any stage \(n\) given no prior selection is \(I_n\) (i.e. (4.5)). Assuming that all pigs in a batch are equal and mutually independent, the number of selected pigs in a batch at a given stage \(n\) given \(k\) remaining pigs at stage \(n\) is binomial distributed, \(B(k, I_n)\). The number of remaining pigs at stage \(n+1\) is then binomial distributed, \(B(k, 1-I_n)\) and the probability of \(l\) remaining pigs at stage \(n+1\) given \(k\) remaining pigs at stage \(n\) is given by:

\[
P_{l,n|k} = \binom{k}{k-l} \cdot I_n^{k-l} \cdot (1-I_n)^l = \binom{k}{l} \cdot I_n^l \cdot (1-I_n)^{k-l} \tag{4.10}
\]

Assuming an infinite uninterrupted sequence of batches (i.e. an infinite time horizon of the problem and infinite duration of the production system) the net present value (NPV) of a section containing \(k\) pigs at stage \(n\) is:
\[ NPV_m(n,k) = -K \cdot PC_n + RNPV_m(n,k) \quad (a) \]

\[
RNPV_m(n,k) = \begin{cases} 
\sum_{l=0}^{k} P_{l,n}[k] \cdot [ESNR_n(k,l) + r \cdot NPV_m(n+1,l)] & n < m \\
ETSNR_n(k) + r^M \cdot NPV_m(1,K) & n = m 
\end{cases}
\]

\[ PC_n = \begin{cases} 
PC, & n = 1 \\
0, & n = 2, \ldots, N 
\end{cases} \quad (4.11) \]

where \( m \) is the terminal marketing stage, \( m = 1, \ldots, N \),

\( n \) is the current marketing stage (and a counter used in the recursion),

\( k \) is the remaining number of pigs (i.e. the batch size) at stage \( n \),

\( ESNR_n(k,l) = (k-l) \cdot ERTM_n - k \cdot EVC_n \) is the expected stage return of the \( k-l \) selected pigs subtracted the variable costs of all \( k \) remaining pigs,

\( ETSNR_n(k) = k \cdot ERTM_n - k \cdot EVC_n \) is the expected stage return of marketing all remaining pigs (i.e. at terminal marketing stage \( m \)),

\( ERSN_n \) is the expected return per pig place given selection,

\( EVC_n \) are the expected variable costs from stage \( n-1 \) to \( n \) (\( EVC_1 = 0 \) and \( EVC_n = VC \) \((E(w_n|v_{n-1}(X'))), n = 2, \ldots, N\)),

\( ERTM_n \) is the expected return given termination,

\( PC \) are the replacement costs/the purchase costs of a new batch,

\( K \) is the standard (and fixed) size of the current and/or a new batch and

\( M \) is the time (measured in stages) from insertion to first marketing stage.

The purchase costs (\( PC \)) include all variable costs associated with the marketing of a batch and insertion (and production) of a new batch, like the cost of new weaners, feeding costs from insertion to first marketing stage (i.e. \( M \) weeks) and labor costs directly associated with the replacement of batches.

The conditional expected stage return of a pig given selection at current stage (and implicitly given no selections at previous stages) is given by:

\[ ERSN_n = \frac{1}{G_n} \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{x,e}(x,e|x_0,x+e) \cdot u(cw_n,cl_n) \cdot cw_n \, dx \, de \]

where \( cw_n \) and \( cl_n \) are scalar elements of \( x \).

The conditional stage return of a pig given termination at current stage (and implicitly given no selections at previous stages) is:

\[ ERTM_n = \frac{1}{H_n} \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{x,e}(x,e|x_{n-1},x+e) \cdot u(cw_m,cl_m) \cdot cw_m \, dx \, de \quad (4.12) \]

Using the solution methods (i.e. Dynamic Programming) of probabilistic problems described
below, equation (4.11) can be optimized directly (i.e. with respect to the terminal marketing stage \( m \)), but a more simple and computational efficient (i.e. precision and time) method is applied as follows.

The two functions \( ETSNR \) and \( ESNR \) are seen to be linear in the argument(s) \( k \) (and \( l \)) implying that:

**Theorem 1.** Let the \( NPV_m \) be given by:

\[
NPV_m = NPVB_m + r^{m+M-1} \cdot NPV_m
\]

\[
NPVB_m = -K \cdot PC + K \cdot \sum_{n=1}^{m-1} r^{n-1} \cdot WSNR_n + K \cdot r^{m-1} \cdot WTSNR_m \quad (4.13)
\]

where \( NPV_m \) is the net present value of the infinite chain of batches,

\( NPVB_m \) is the net present value of a single batch,

\( WSNR_n = G \cdot ERSM - H \cdot EVC \) is the expected stage net return per pig place of selecting and marketing pigs at stage \( n \), weighted with the probability of selection at (and the probability of no selection prior to) stage \( n \) and

\( WTSNR_m = H_m \cdot (ERTM_m - EVC_m) \) is the expected terminal stage net return per pig place of the terminal marketing at stage \( m \), weighted with the probability of no selection prior to stage \( m \).

Then, for any \( m = 1, \ldots, N \), \( K \in \mathbb{R} \):

\[
NPV_m = NPV_m (1, K)
\]

**Proof:**

Expanding the recursions in (4.11), \( NPV_m (1, K) \) is:

\[
NPV_m (1, K) = \]

\[
NPV_m = -K \cdot PC + \sum_{k_1=0}^{K} p_{k_1} \cdot [ESNR_1(K, k_1)] + r \cdot \sum_{k_2=0}^{k_1} p_{k_2} \cdot [ESNR(k_1, k_2)] + \ldots
\]

\[
+ r \cdot \sum_{k_{m-2}}^{k_{m-1}} p_{k_{m-2}} \cdot [ESNR(k_{m-2}, k_{m-1})] + r \cdot [ETSNR(k_{m-1}) + r^{M} \cdot NPV_m] \ldots]
\]

(4.14)

Now let the two functions \( \varphi_n \) and \( \kappa_n \) be given by:
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\[
\Phi_n(k) = \sum_{l=0}^{k} p_{l,n,k} \cdot ESNR(k,l) \quad (a)
\]

\[\kappa_n(\omega) = \begin{cases} 
\sum_{k_1=0}^{K} [p_{k_1,1}] \sum_{k_2=0}^{k_1} [p_{k_2,2}] \cdots \sum_{k_{n-2}=0}^{k_{n-3}} [p_{k_{n-2},n-2}] \cdot \omega(k_{n-1}) \cdots ] 
\end{cases} \quad n = 2, \ldots, N \tag{4.15}
\]

(\omega(K),

\[\kappa_n(\omega) = \begin{cases} 
\sum_{k=0}^{K} [p_{k,n}] \cdot \omega(k) \quad (a)
\end{cases} \quad (4.16)
\]

Then by assuming that the argument (i.e. the function \(\omega\)) of \(\kappa\) in (4.15) is a linear function in its own argument, by applying (4.16), by induction over \(n\) and by applying (4.6), \(\kappa\) is given by:

\[\kappa_n(\omega) = \begin{cases} 
\sum_{k_1=0}^{K} [p_{k_1,1}] \sum_{k_2=0}^{k_1} [p_{k_2,2}] \cdots \sum_{k_{n-2}=0}^{k_{n-3}} [p_{k_{n-2},n-2}] \cdot \omega(k_{n-1}) \cdots ] 
\end{cases} \quad n = 1, \ldots, N \tag{4.17}
\]

and \(\phi\) is given by:

\[\Phi_n(k) = k \cdot I_n \cdot ERSM_n - k \cdot EVC_n \quad k \cdot (I_n \cdot ERSM_n - EVC_n) \tag{4.18}\]

\(\phi\) is seen to be linear in its argument \((k)\) and then by removing parentheses in (4.14) and substituting (4.15.a) and (4.17) in (4.14), \(NPV_m^{*}\) is given by:

\[\begin{align*}
NPV_m^{*} &= -K \cdot PC + \kappa_1(\phi_1) + r \cdot \kappa_2(\phi_2) + r^2 \cdot \kappa_3(\phi_3) + \ldots \\
&+ r^{m-2} \cdot \kappa_{m-1}(\phi_{m-1}) + r^{m-1} \cdot [\kappa_m(ETSNR_m) + r^M \cdot NPV_m^{*}] \\
&= -K \cdot PC + H_1 \cdot \phi_1(K) + r \cdot H_2 \cdot \phi_2(K) + r^2 \cdot H_3 \cdot \phi_3(K) + \ldots \\
&+ r^{m-2} \cdot H_{m-1} \cdot \phi_{m-1}(K) + r^{m-1} \cdot [H_m \cdot ETSNR_m(K) + r^M \cdot NPV_m^{*}] \\
\tag{4.19}
\end{align*}\]

Substituting (4.18) in (4.19), applying (4.5), adding a summation sign and splitting the equation into two then gives (4.13).

Q.E.D.
Optimizing (4.13) with respect to \( m \) gives the optimal terminal marketing stage given constant and inflexible (i.e. same terminal marketing stage for all batches) weaner supply. The criterion appears to be the basic optimization criterion of the traditional replacement theory as described by e.g. J. S. Taylor in 1924, (cf. Preinrich), Preinrich (1940) and Rasmussen (1976), with the batch representing the asset, \( K:\text{PC} \) representing the purchase costs of the asset, \( K:\text{WSNR} \) representing the net return per time step and \( WTSNR \) representing the terminal value of the asset.

Assuming that current and future batches are equal (which is a special case of the general replacement problem, see e.g. Rasmussen, 1976), \( NPV_m \) may as well be optimized by maximizing the average net return per pig place per week:

\[
ANR^* = \max_{m \in \{1, \ldots, M\}} \left\{ \frac{NPVB_m}{K} \cdot \frac{i}{1 - (1 + i)^{(m+M-1)}} \right\} \quad (a)
\]

\[
NPV^* = K \cdot \frac{ANR^*}{i} \quad (b)
\]

where \( ANR^* \) is the optimal average net return per pig place per week, calculated as an annuity and

\( NPV^* \) is the optimal net present value of a section (or the infinite sequence of batches).

Inserting (4.12) and (4.12) in (4.20) then gives the optimization criteria for the optimal marketing stage given constant and inflexible weaner supply:

\[
ANR^* = \max_{m \in \{1, \ldots, M\}} \left\{ \left\{ -PC + \sum_{n=1}^{m-1} r^{m-1} \cdot [G_n \cdot ERSM_n - H_n \cdot VC_n (E(W_m | \psi_{m-1}(X))) + r^{m-1} H_n \cdot ERTM_m - VC_m (E(W_m | \psi_{m-1}(X))) \right\} \cdot \frac{i}{1 - (1 + i)^{(m+M-1)}} \right\} \quad (4.21)
\]

(4.21) appears to be independent of the actual batch size, \( K \) (or the distribution of the batch size) at a given stage. The equation contains no recursive elements and compared to (4.11) (solved by Probabilistic Optimization, see below) the computational burdens are in most cases (i.e. when \( K \) is large) drastically reduced.

Optimization criteria (4.21) and (4.20) are in the following referred to as Deterministic Optimization (DO) as the calculations are based on expectations of distributions (of batch sizes at different stages) rather than directly on the distributions as in (4.11).

It should be noted that if a batch is not (immediately) replaced by a new batch the same criteria as of individual pigs (i.e. (4.2)) apply: The batch should be terminated if the marginal net return of postponing the marketing 1 stage is negative.

4.2.5 Terminal marketing. Constant, but flexible weaner supply.

Let the set of feasible marketing decisions be given by: \( \Xi = \{ \text{“K”, “M”} \} \), where “K” is interpreted
as “keep the remainder of the batch one more week” and “M” as “market the remainder of the batch now”. A marketing state is defined as the tuple of a marketing stage \( \mathcal{E} \) and a batch size \( \mathcal{F} \). Then (4.10) expresses the probability of the transition (i.e. the transition probability) from state \((n, k)\) to state \((n+1, l)\), for \( n = 1, \ldots, N-1 \), \( k = 0, \ldots, K \) and \( l = 0, \ldots, k \). A policy is specified as a finite mapping from a marketing state into a decision: \( \mathcal{Y}: \mathcal{E} \times \mathcal{F} \rightarrow \mathcal{E} \). Finally a decision set is specified as a finite mapping from a marketing stage into a set of decisions and is defined as:

\[
D_n = \begin{cases} 
\{"K", M\}, & n = 1, \ldots, N-1 \\
\{"M"\}, & n = N
\end{cases}
\]

In (4.11) the argument \( m \) may be interpreted as a very inflexible policy: “Always perform the terminal marketing at stage \( m^\prime \)”. Allowing the terminal marketing stage to differ from batch to batch (i.e. flexible terminal marketing stage) (4.11) might be redefined as:

\[
NPV_n(k, P) = -K \cdot PC_n + v_n(k, P(n, k)) \quad (a)
\]

\[
v_n(k, d) = \begin{cases} 
\sum_{l=0}^{k} P_{kl} [ESNR_n(k, l) + r \cdot NPV_{n+1}(l, P)] & d = "K" \\
ETSNR_n(k) + r^{M-1} \cdot NPV_1(K, P) & d = "M"
\end{cases} \quad (b)
\]

where \( P \) is any feasible policy, \( P \in \mathcal{Y} \), and with the optimization criterion:

\[
NPV^*(k) = -K \cdot PC_n + \max_{P \in \mathcal{Y}} \{v_n(k, P(n, k))\}
\]

Applying Bellman’s Principle of Optimality (Bellman, 1957; Kure, 1995) the global problem (ie of optimizing policies) is decomposed into a hierarchy of local (sub-)problems (of optimizing decisions):

\[
\Psi_n(k) = -K \cdot PC_n + \max_{d \in D_n} \{\omega_n(k, d)\} \quad (a)
\]

\[
\omega_n(k, d) = \begin{cases} 
\sum_{l=0}^{k} P_{kl} [ESNR_n(k, l) + r \cdot \Psi_{n+1}(l)] & d = "K" \\
ETSNR_n(k) + r^{M-1} \cdot \Psi_1(K) & d = "M"
\end{cases} \quad (4.22)
\]

The computational benefits of applying the decomposed criterion (also known as Dynamic Programming) are in most cases of huge dimensions and in many situations it will turn (real time) unsolvable problems into solvable problems.

The optimal average net return per pig placed per week is given by:
$$\text{ANR}^* = \frac{\Psi_1(K)}{K} \cdot i \quad (4.23)$$

Optimization criterion (4.23) will be referred to as \textit{Probabilistic Optimization} (PO).

For a given policy it may be interesting to know the distribution of states given that policy. This distribution can in general be calculated by use of Influence Diagrams or by an adjusted version of RDP as defined by Kure (1995), but in at least one special situation a more simple method may be applied. If the policy is to keep the batch at all stages, the number of pigs remaining at stage \(n\) will be binomial distributed, \(B(K, 1-H_n)\) and the probability of \(l\) remaining pigs at stage \(n\) is:

$$p_{l,n} = \binom{K}{l} \cdot (1-H_n)^n \cdot H_n^{(k-l)}$$

### 4.2.6 Terminal marketing. Varying and flexible weaner supply

Let a stochastic flexible weaner supply be represented by the stochastic variable \textit{Weaner} and let the distribution of \textit{Weaner} be approximated by a normal distribution:

$$p_l = \begin{cases} 
F_{\text{Weaner}}(l + \frac{1}{2}), & l = 0 \\
F_{\text{Weaner}}(l + \frac{1}{2}) - F_{\text{Weaner}}(l - \frac{1}{2}), & l = 1, \ldots, K \\
1 - F_{\text{Weaner}}(l - \frac{1}{2}), & l = K 
\end{cases}$$

where \(p_l\) is the approximated probability function and \(F_{\text{Weaner}}\) is the density mass function of \textit{Weaner} \(\sim N (\mu, \sigma^2)\).

And let the expected batch size \((K')\) and the expected surplus of weaners \((K'')\) be given by:

$$K' = (1-F_{\text{Piglet}}(K+\frac{1}{2})) \cdot [E(Piglet|Piglet>K+\frac{1}{2}) - (K+\frac{1}{2})]$$

$$K'' = \sum_{l=1}^{K} p_l \cdot l$$

The deterministic optimization criterion (ie given constant terminal marketing stage) is then given by (4.13) with \(NPVB_m\) redefined as:

$$NPVB_m = -K \cdot FPC - K' \cdot LEP + K'' \cdot [-VPC + \sum_{n=1}^{m-1} r^{n-1} \cdot ESNR_n + r^{m-1} \cdot ETSNR_m]$$

$$PC = FPC + VPC$$

where \(FPC\) are the fixed purchase costs (i.e. independent on batch size),
\[ LEP \] is the loss per weaner in excess and \\
\[ VPC \] are the variable purchase costs (i.e. dependent on the batch size).

and in the case of probabilistic optimization (i.e. changeable weaner supply) \( \Psi_n \) in (4.22) is redefined as:

\[
\Psi_n(k) = \begin{cases} 
K \cdot FCP & - K' \cdot LEP + \sum_{l=1}^{K} p_l \cdot I[- VPC + \max_{d \in D_n} (\omega_n(l, d))], \\
\max_{d \in D_n} (\omega_n(k, d)) & n = 2, \ldots, N
\end{cases}
\]  

(4.24)

### 4.2.7 Probabilistic Optimization

The functional equations (4.22) and (4.24) can be solved by a number of Dynamic Programming (DP) solution methods. Bellman (1957) first introduced and formalized the concept of DP as a backward multistage problem solving technique of sequential decision problems (Nemhauser, 1966). Howard (1960) named this technique as Value Iteration (VI) and introduced a new technique, Policy Iteration (PI), which in special situations (infinite time horizon and “small” state spaces) proved to be very efficient (numerically, concerning precision and computing time) compared to VI. Kristensen (1988) introduced a new method, Hierarchic Markov Processes (HMP), as a hybrid of VI and PI, utilizing the best features of the two methods: handling of large state spaces and fast convergence respectively. The method increased the numerical efficiency compared to VI and PI and has been successfully applied to large–state space problems with more than 6,000,000 states (Houben, 1995). Shenoy (1992) demonstrated how the general principle of DP is utilized in Influence Diagrams and Valuation Networks. In chapter 5 (and in Kure, 1995) it is demonstrated how the introduction of recursive solution techniques (Recursive Dynamic Programming (RDP)) might increase the computational efficiency and contribute to the theoretical generality of DP with and without stochastic (Markov) processes.

The optimization method applied in this presentation is described as a mixture of HMP with 1 main process (see Kristensen, 1988) and RDP; the VI–steps of the HMP are substituted by RDP. The iterative method is as follows:

![Diagram](image_url)

**Figure 4.4.** Probabilistic optimization, Constant weaner supply. Nodes represent states.
4.2 Material and methods

1. Set an initial “guess”, \( NP\bar{V}_0 \) of the solution of the functional equation (4.22) or (4.24). Set \( t = 1 \).

2. Optimize the policy by applying RDP given the current value of \( NP\bar{V}_{r-1} \) (i.e. solve/calculated (4.22) or (4.24), with \( \Psi_t(K) \) in (4.22.b) substituted by the constant value \( NP\bar{V}_{r-1} \)). The value of the solution is termed \( \Psi_t(K)^o \). Calculate the “average discount factor”, \( q_t \) (is automatically calculated using the general RDP method, with discounting, see chapter 5).

3. If \( |\Psi_t(K)^o - NP\bar{V}_t| > \epsilon \), where \( \epsilon \) is a predefined convergence limit (\( \epsilon \) > the numerical precision of the computing device).

   then: Recalculate the net present value given the optimal value \( \Psi_t(K)^o \) and the “average discount factor” \( q_t \) from 2.: 
   \[ NP\bar{V}_t = NP\bar{V}_{r-1} + (\Psi_t(K)^o - NP\bar{V}_{r-1})/q_t. \]
   Set \( t = t + 1 \). Goto step 2.

   else: End the iteration process.

Step 2 of the algorithm – in the situation of a constant, but flexible weaner supply with 2 pigs per batch and 5 marketing stages – is shown graphically in fig. 4.4 (stage 4 and parts of stage 3 are not shown). The graph is traversed recursively from left to right following the directed edges as described in chapter 5 and the dashed directed edge from right to left represent the step 1 and 3 in the iteration. Decision nodes represent the decision of either “Keep” or “Market” all remaining pigs in the batch, while chance nodes represent the stochastic process of selecting and marketing individuals, with transition probabilities as defined by (4.10). In the initial node, at stage 1, the option is to either market or keep the 2 remaining pigs, while the only option in the nodes at the terminal stage, 5, is to market all remaining pigs.

In fig. 4.5 the same situation, but with varying weaner supply is shown. Compared to fig. 4.4 a chance node representing the stochastic weaner supply (at stage 0, i.e. prior to the first marketing stage) has been added and the graph extended/changed accordingly. However, same optimization algorithm is used to solve both problems.

4.2.8 Model testing

In order to test and examine the proposed models and methods a sample feeder pig finishing operation is introduced. Parameters of the animal growth model are estimated based partly on simulated data using the simulation model developed and described by Jørgensen (1993) and

![Figure 4.5. Probabilistic optimization. Varying weaner supply. Nodes represent states.](image-url)
Jørgensen & Kristensen (1995); partly on a data set from The Danish Slaughter Houses (1011 samples from an experiment first published by Udesen, 1993) in which only 5–10% of the pigs have been selected for marketing and on a comprehensive (i.e. many longitudinal observations) data set (189 samples) from a currently unpublished experiment at the Royal Agricultural and Veterinary University, Copenhagen (see also chapter 2, 3). Prices paid by the packer are as shown in fig. 4.1. All necessary model parameters (except prices) are with their default values shown in the appendix.

The sample feeder pig finishing operation (with default parameter values) is used for basic optimizations and for examining and comparing the different selection criteria and optimization methods proposed above. In order to test the sensitivity of the methods a range of model parameters (in particular the parameters of the animal growth model) have been changed from the default values. In the optimizations of the selection of individuals (e.g. when calculating $ER_{I_n}$ in (4.9)) the sampling size, $Z$, was 5,000 while in the calculations of probability of selection etc. (e.g. when calculating $H_n$ in (4.3)) the sampling size was increased to 200,000 in order to get exact statistics on model performances/output. The results of these analysis’ are presented in the Results section below.

4.3 Results

Selected results are presented below. It should be noted that results are based on simulated data and should not be interpreted without precautions.

4.3.1 Selection of individual pigs

4.3.1.1 Selection based on observed live weight

The basic results of selection on observed live weight at stage 1 and 3 (i.e. $W_1$’ and $W_3$’) are shown in fig. 4.6. The marginal net return is positive for $w_1$’ $< 93.9$ kg and for $w_1$’ $> 107$ and negative for $93.9 < w_1$’ $< 107$ kg; only pigs in the interval from 93.9 kg to 107 kg live weight should be marketed. The probability of an observed live weight above 107 kg is, as indicated by the marginal density of $W_1$’ in the figure, very little and therefore the results will not be significantly

![Figure 4.6](image-url)

**Figure 4.6.** Marginal density of observed live weight $w$’ and expected marginal net return (MNR) conditioned on observed live weight at stage 1 and 3. $E_{w} - 0.5^2$. $Z - 5,000$. 

influenced by omitting this upper interval in the selection criteria.

The drop in MNR at approximately 118 kg live weight is caused by a severe price penalty on heavy pigs (carcass base price is reduced from 9.20 to 7.30 DKK per kg at 102 kg carcass weight). The optimal threshold live weights of all 4 stages (stage 5 is omitted as all remaining pigs will be marketed in any case at this stage) are shown in table 4.2. It should be noted how the threshold weights decreases as the marketing stage number increases.

In fig. 4.7 the marginal net returns from stage 1 to stage 2, 3, 4 and 5 (i.e. MNR₁ (1), MNR₁ (2), MNR₁ (3) and MNR₁ (4) respectively) are shown (the graph of MNR₁ (1) in fig. 4.7 represents the same data as the graph of MNR₁ (1) in fig. 4.6). MNR₂ (2), MNR₃ (3) and MNR₄ (4) are at least in the most important (and likely) interval from 93.9 kg to approximately 103 kg less than MNR₁ (1) or at least less than 0. For pigs with an observed live weight of more than approx. 103 kg it will be beneficial to keep the pigs until marketing stage 4, but again the probability as shown by the marginal density function of this heavy weight and corresponding high growth rate is relatively little. In general the assumption of decreasing marginal net return over time is satisfied within the most important (and likely) weight intervals. An analysis of stage 2, 3 and 4 gives similar results.

4.3.1.2 Selection based on observed live weight and carcass leanness

The results of conditioning the expected marginal net return (of postponing the marketing 1 stage) on observed live weight and observed carcass leanness using (4.8) are shown in fig. 4.8. For cl₁ ′ = 0.598 (i.e. the expected carcass leanness at stage 1, see the appendix) the graph of MNR₁ (1) in fig. 4.6 may be recognized. The interesting and important issue is that the intersection between MNR₁ for 80 < w₁ ′ < 104 and the zero plane in fig. 4.8 is approximately a straight line. A selection criterion based on this linear relationship is given by (4.25) and functional estimates of the relationships for stage 1 to 4 are shown graphically in fig. 4.9. The ‘+’ and ‘−’ signs

<table>
<thead>
<tr>
<th>n</th>
<th>w'</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>93.9</td>
</tr>
<tr>
<td>2</td>
<td>93.6</td>
</tr>
<tr>
<td>3</td>
<td>93.5</td>
</tr>
<tr>
<td>4</td>
<td>91.9</td>
</tr>
</tbody>
</table>

Table 4.2. Optimal threshold values (w') at stage (n) 1 to 4. E_w = 0.5^2. Z = 5,000.

Figure 4.7. Marginal prior density (f(w₁')) of live weight (w₁') at stage 1 and marginal net return (MNR₁) from stage 1 to 2,3,4 and 5. E_w = 0.5^2. Z = 5,000.
\[
\zeta_n(x^*) = \begin{cases} 
(w_n^* > w_n^{**}), & n = 1, \ldots, N-1 \\
\text{TRUE}, & n = N 
\end{cases} \quad (a)
\]

\[
w_n^* = \beta_0 + \beta_1 \cdot c_l^* \quad (b)
\]

indicate positive and negative values of \( MNR_n \) (1): If \( w_n^* > w_n^{**} \) for a specific pig then \( MNR_n \) (1) is negative and the decision is whether to market the pig at current stage or to keep the pig (at least) until next stage.

**Figure 4.8.** Expected net marginal return at stage 1 (MNR) conditioned on observed live weight (W') and observed carcass leanness (CL') at stage 1. \( E_w = 0.5^2 \) and \( E_{cl} = 0.002^2 \). \( Z = 5,000 \).

**Figure 4.9.** Mixed selection criterion: Selection on observed live weight (W') as a function of observed carcass leanness (CL') at stage \( n = 1, 2, 3, 4 \): Select if \( w^* > \beta_0 + \beta_1 \cdot c_l^* \). \( E_w = 0.5^2 \) and \( E_{cl} = 0.002^2 \). \( Z = 5,000 \).
4.3 Results

4.3.2 Terminal marketing

4.3.2.1 Constant weaner supply

The 4 selection criteria (SC’s) as described and assessed in previous sections have been applied to the two different optimization criteria (Deterministic Optimization (DO, (4.21)) and Probabilistic Optimization (PO, (4.23))) in order to find the optimal terminal marketing stages and the optimal expected average net returns. The basic results of these calculations are shown in table 4.3. With selection criterion I (i.e. no selection) and with DO the optimal terminal marketing stage is 3 (assuming same costs of marketing/insertion for all stages – no additional costs associated with changing the weaner supply) and the optimal average net return per pig place per week (ANR\textsuperscript{1}) is 6.15 DKK. With selection criterion II (selection on observed live weight) the ANR\textsuperscript{1} is increased by 0.31 DKK (or 0.31(1−(1+i)^{52})/i = 15.72 DKK/year) to 6.46 DKK. The optimal marketing stage is 4, but marketing at stage 3 is only slightly inferior (0.01 DKK). The difference in ANR\textsuperscript{1} between criterion II and III (selection on observed live weight and observed carcass leanness) appears to be relatively little (0.01 DKK), while the difference between criterion IV (perfect information) and III is significantly larger: 0.07 DKK (or 3.55 DKK per pig place per year). The difference between ANR\textsuperscript{1} of different criteria increases as m (and the fraction of selected pigs) increases and the loss of executing the terminal marketing too late (e.g. at stage 5 instead of stage 3) is largest (1.82 DKK per pig place per week) when not selecting at all (i.e. criterion I).

The ANR\textsuperscript{1} of PO depends greatly on the batch size: values decrease as batch sizes increase and the values converge towards the results of DO. The difference between different selection criteria are of same magnitude as of DO.

With DO the likelihoods of selection prior to the optimal stage (i.e. G\textsubscript{m} * in table 4.4) is 47.1% for SC II, 47.6% for SC III and 51.3 for SC IV, implying that in all 3 cases approximately 50% of all pigs should be selected and marketed prior to the terminal marketing at stage 4. The average fraction of Top Grade pigs (i.e. TG\textsuperscript{*}: pigs with a carcass weight between 64 and 78 kg, which as shown in fig. 4.1 are paid the highest base price) is largest for SC III and approximately 2 %–point smaller for SC IV. Average carcass leanness (i.e. c\textsubscript{L} \textsuperscript{*}) is lower (approximately 0.1 %–point) for SC II and III than for SC I, but the same for SC I and SC IV. Average price (Price\textsuperscript{*}) is almost the

<table>
<thead>
<tr>
<th>Selection criteria (SC)</th>
<th>DO</th>
<th>PO</th>
</tr>
</thead>
<tbody>
<tr>
<td>I (None)</td>
<td>m = 1  m = 2  m = 3  m = 4  m = 5</td>
<td>K = 1  K = 2  K = 4  K = 8  K = 20  K = 100</td>
</tr>
<tr>
<td>II (w)</td>
<td>4.66  5.90  6.15  5.65  4.33</td>
<td>6.15  6.15  6.15  6.15  6.15  6.15</td>
</tr>
<tr>
<td>III (w, ch)</td>
<td>4.66  5.96  6.45  6.46  6.13</td>
<td>6.58  6.54  6.50  6.49  6.48  6.47</td>
</tr>
</tbody>
</table>

Table 4.3. Constant weaner supply. Average weekly net return per pig place (ANR) of Deterministic and Probabilistic Optimization (DO and PO) given 4 different selection criteria (SC, see text for explanation and definition of criteria). Optimal ANRs of DO are indicated by a shaded area. All ANRs of PO are optimal. \(E_{\text{w}} = 0.5^2\) and \(E_{\text{cl}} = 0.002^2\). Z = 200,000.
### Table 4.4. Constant weaner supply. Summary of results of Deterministic Optimization given 4 different selection criteria (SC). Optimal stage (m), optimal ANR (ANR*), selection likelihood (G_i) at all 5 stages, accumulated selection likelihood at m (G_m*), likelihood of no selection prior to stage m (H_m*), and weighted average (of all pigs and given terminal marketing at stage m) of carcass weight (\( \overline{cw} \)), Top Grade likelihood (\( \overline{TG} \)), carcass leanness (\( \overline{TR} \)) and price (\( \overline{Price} \)). \( E_w = 0.5^2 \) and \( E_{cl} = 0.002^2 \). Z = 200,000.

<table>
<thead>
<tr>
<th>SC</th>
<th>m*</th>
<th>ANR* (DKK)</th>
<th>G_n</th>
<th>G_m*</th>
<th>H_m*</th>
<th>( \overline{cw} ) (kg)</th>
<th>( \overline{TG} ) (%)</th>
<th>( \overline{TR} ) (%)</th>
<th>( \overline{Price} ) (DKK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>3</td>
<td>6.15</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>73.37</td>
<td>68.33</td>
<td>58.99</td>
<td>10.05</td>
</tr>
<tr>
<td>II</td>
<td>4</td>
<td>6.46</td>
<td>0.052</td>
<td>0.164</td>
<td>0.255</td>
<td>74.30</td>
<td>87.24</td>
<td>58.87</td>
<td>10.14</td>
</tr>
<tr>
<td>III</td>
<td>4</td>
<td>6.47</td>
<td>0.053</td>
<td>0.167</td>
<td>0.256</td>
<td>74.26</td>
<td>87.66</td>
<td>58.88</td>
<td>10.15</td>
</tr>
<tr>
<td>IV</td>
<td>4</td>
<td>6.54</td>
<td>0.056</td>
<td>0.182</td>
<td>0.275</td>
<td>74.20</td>
<td>85.55</td>
<td>58.99</td>
<td>10.15</td>
</tr>
</tbody>
</table>

The same for SC II, III and IV, but approximately 0.10 DKK lower for SC I. The average carcass weight (\( \overline{cw} \)) does not differ much between SC II, III and IV, but the variance on \( \overline{cw} \) (not shown in the table) is significantly higher for SC IV than for SC II and III (e.g. 32.32, 12.47 and 11.97 kg² respectively of pigs marketed at stage 5).

In Table 4.5 an example of more detailed results of Probabilistic Optimization is shown. For all states (i.e. combinations of remaining pigs and marketing stage) the optimal decisions are shown and the table (i.e. the mapping from state to decision) represents the optimal policy as previously defined. The optimal expected average net returns (ANR*) of following the optimal policy (from any feasible state) are shown in the 2nd section of the table. As a new batch is assumed to always contain \( K \) pigs only one state is optimized at stage 1. The distributions of states at each stage are shown in the 3rd section of table 4.5 and it should be noted that the prior probability of termination (i.e. the probability of states for which the optimal decision is “M”) at stage 3 is 54.2%, leaving a 45.8% probability of termination at succeeding stages (i.e. at stage 4).

### Table 4.5. Constant weaner supply. Example of output of Probabilistic Optimization. Optimal decisions (d*), optimal ANR (ANR*) and probabilities of states given d* = “K” for all states (i.e. combinations of state (n) and remaining pigs (k)). “Keep” decisions indicated by shaded area. \( K = 8, SC = 11 \) and \( E_w = 0.5^2 \). Z = 200,000.

<table>
<thead>
<tr>
<th>k</th>
<th>n = 1</th>
<th>n = 2</th>
<th>n = 3</th>
<th>n = 4</th>
<th>n = 5</th>
<th>d*</th>
<th>ANR* (DKK)</th>
<th>P_{kn}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“M”</td>
<td>5.86</td>
<td>.000</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“M”</td>
<td>5.94</td>
<td>.000</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“M”</td>
<td>6.02</td>
<td>.000</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“M”</td>
<td>6.10</td>
<td>.000</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“K”</td>
<td>6.19</td>
<td>.000</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“K”</td>
<td>6.27</td>
<td>.006</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“K”</td>
<td>6.35</td>
<td>.055</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“K”</td>
<td>6.44</td>
<td>.287</td>
</tr>
<tr>
<td>8</td>
<td>“K”</td>
<td>“K”</td>
<td>“K”</td>
<td>“M”</td>
<td>“M”</td>
<td>“M”</td>
<td>6.49</td>
<td>.651</td>
</tr>
</tbody>
</table>

The same for SC II, III and IV, but approximately 0.10 DKK lower for SC I. The average carcass weight (\( \overline{cw} \)) does not differ much between SC II, III and IV, but the variance on \( \overline{cw} \) (not shown in the table) is significantly higher for SC IV than for SC II and III (e.g. 32.32, 12.47 and 11.97 kg² respectively of pigs marketed at stage 5).

In Table 4.5 an example of more detailed results of Probabilistic Optimization is shown. For all states (i.e. combinations of remaining pigs and marketing stage) the optimal decisions are shown and the table (i.e. the mapping from state to decision) represents the optimal policy as previously defined. The optimal expected average net returns (ANR*) of following the optimal policy (from any feasible state) are shown in the 2nd section of the table. As a new batch is assumed to always contain \( K \) pigs only one state is optimized at stage 1. The distributions of states at each stage are shown in the 3rd section of table 4.5 and it should be noted that the prior probability of termination (i.e. the probability of states for which the optimal decision is “M”) at stage 3 is 54.2%, leaving a 45.8% probability of termination at succeeding stages (i.e. at stage 4).
The number of iterations in the HMP algorithm ranged in all optimizations from 3 to 5.

In order to test the sensitivity and robustness of the model a wide range of model parameters have been changed from their default values and the model output compared to the default results as presented above.

The effects of changing prices and expectations and variances on animal traits are shown in table 4.6. The optimal threshold weights are in general quite unaffected by the changes in the parameters. At stage 1 the change in $w^*$ is no more than ±0.8 kg, while the change at stage 4 is within the range of −1.9 to 1.0 kg (all extremes caused by the change in carcass base price).

The optimal average net returns per pig place per week ($ANR^*$) are as indicated by the shaded areas in table 4.6 more affected by changes in carcass base price and expectations of traits than by changes in variances of traits (and by changes in feed price). A 10% increase in carcass base price will (all other parameters kept equal) result in a 81.7% (i.e. from 6.46 to 11.74 DKK per week) increase in $ANR^*$, while a 10% decrease will result in a 78.8% decrease in $ANR^*$. An increase in variances by 20% increases the advantage of SC III compared to SC II from 0.01 (see table 4.3 and 4.4) to 0.03 (or 0.0254 with 4 digits precision) DKK per week (or 1.29 DKK per pig place per year or a NPV of 27.06 DKK per pig place). A large decrease (50%) in the variances on traits will for SC II cause a 4.5% increase in $ANR^*$ and the difference between SC II and IV (i.e. perfect information) is reduced from 0.08 DKK (i.e. table 4.3 and 4.4) to 0.02 DKK per week.

### Table 4.6. Constant weener supply. Model output given partial changes ($\Delta$) in major model parameters and different selection criteria (SC). Optimal weight ($w^*$), average net return ($ANR^*$) (optimal values indicated by a shaded area), average of (all pigs and given terminal marketing at optimal marketing stage n') carcass weight ($c_{\text{w}}$) and average price ($\text{Price}^*$). $OM = DO$, $E_w = 0.5^2$, and $E_{cl} = 0.002^2$. $Z = 200,000$.

<table>
<thead>
<tr>
<th>Model parameter</th>
<th>$\Delta$</th>
<th>$\text{SC}$</th>
<th>$w^*$ (kg)</th>
<th>$ANR^*$ (DKK)</th>
<th>$c_{\text{w}}$</th>
<th>$\text{Price}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$n-1$</td>
<td>$n-2$</td>
<td>$n-3$</td>
<td>$n-4$</td>
<td>$m-1$</td>
</tr>
<tr>
<td>---</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$m-1$</td>
</tr>
<tr>
<td>Price</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+10%</td>
<td></td>
<td>II</td>
<td>92.9</td>
<td>93.6</td>
<td>93.5</td>
<td>91.9</td>
</tr>
<tr>
<td>-10%</td>
<td></td>
<td>II</td>
<td>93.1</td>
<td>92.8</td>
<td>92.7</td>
<td>90.0</td>
</tr>
<tr>
<td>Feed price</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+10%</td>
<td></td>
<td>II</td>
<td>93.5</td>
<td>93.1</td>
<td>93.0</td>
<td>91.0</td>
</tr>
<tr>
<td>-10%</td>
<td></td>
<td>II</td>
<td>94.3</td>
<td>94.0</td>
<td>93.9</td>
<td>92.6</td>
</tr>
<tr>
<td>$E(H)$, $E(CW)$, $E(CL)$</td>
<td>5%</td>
<td>II</td>
<td>94.2</td>
<td>93.8</td>
<td>93.7</td>
<td>92.3</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>II</td>
<td>93.7</td>
<td>93.3</td>
<td>93.2</td>
<td>91.6</td>
</tr>
<tr>
<td>$V(H)$, $V(CW)$, $V(CL)^3$</td>
<td>+10%</td>
<td>II</td>
<td>93.9</td>
<td>93.6</td>
<td>93.5</td>
<td>92.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>III</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>+20%</td>
<td>II</td>
<td>93.8</td>
<td>93.5</td>
<td>93.4</td>
<td>92.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>III</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>II</td>
<td>94.0</td>
<td>93.7</td>
<td>93.5</td>
<td>92.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IV</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>-50%</td>
<td>II</td>
<td>94.0</td>
<td>93.6</td>
<td>93.5</td>
<td>92.2</td>
</tr>
</tbody>
</table>

$^3$ Correlations are unchanged.
Table 4.7. Constant weaner supply. Model output given reductions (Δ) in absolute correlations between traits at same stage (ρ), between traits at different stages (ρ_{n,m}) and between CL and all traits at different stages (ρ_{CL,n,m}) and given different selection criteria (SC). Optimal weight (w'), average net return (ANR) (optimal values indicated by a shaded area), average (of all pigs and given terminal marketing at optimal marketing stage m') carcass weight (cw') and average price (Price). \(OM - DO\) and \(E_{w} - 0.5\). \(Z - 200.000\).

<table>
<thead>
<tr>
<th>Model parameter</th>
<th>Δ</th>
<th>SC</th>
<th>w' (kg)</th>
<th>ANR (DKK)</th>
<th>cw' (kg)</th>
<th>Price (DKK)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>n - 1</td>
<td>n - 2</td>
<td>n - 3</td>
<td>n - 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>II</td>
<td>93.9</td>
<td>93.6</td>
<td>93.5</td>
<td>91.9</td>
</tr>
<tr>
<td>[ρ_{n,m}]</td>
<td>-1%</td>
<td>II</td>
<td>93.9</td>
<td>93.5</td>
<td>93.3</td>
<td>91.7</td>
</tr>
<tr>
<td></td>
<td>-5%</td>
<td>II</td>
<td>93.3</td>
<td>93.0</td>
<td>92.8</td>
<td>91.1</td>
</tr>
<tr>
<td>[ρ_{n,m}] and</td>
<td>-1%</td>
<td>II</td>
<td>94.1</td>
<td>93.6</td>
<td>93.4</td>
<td>91.6</td>
</tr>
<tr>
<td>[ρ_{n}]]</td>
<td></td>
<td>IV</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>[ρ_{CL,n,m}]</td>
<td>-1%</td>
<td>II</td>
<td>94.5</td>
<td>93.6</td>
<td>93.0</td>
<td>90.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IV</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>[ρ_{CL,n,m}]</td>
<td>-1%</td>
<td>II</td>
<td>93.9</td>
<td>93.6</td>
<td>93.5</td>
<td>91.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IV</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>[ρ_{CL,n,m}]</td>
<td>-5%</td>
<td>II</td>
<td>93.9</td>
<td>93.6</td>
<td>93.5</td>
<td>91.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IV</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>[ρ_{CL,n,m}]</td>
<td>-50%</td>
<td>II</td>
<td>93.9</td>
<td>93.5</td>
<td>93.5</td>
<td>91.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IV</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

The results of changing the correlations (i.e. reducing the absolute values) between traits (or the covariances in the (co)variance matrix) in the animal model are shown in Table 4.7. With selection criteria (SC) IV (i.e. perfect information) reductions in correlations between traits at same marketing stage (ρ_{n}) as well as between traits at different marketing stages (ρ_{n,m}) contribute to an increase in ANR and (ANR'). More pigs are likely to be marketed at a heavy weight as ρ_{n,m} is decreased and the likelihood of heavy and fat pigs is reduced as the absolute value of the negative correlation between carcass weight and carcass leanness is reduced. When selecting on observed live weight (i.e. SC II) reductions in ρ_{n,m} are increasing ANR while reductions in ρ_{n} are decreasing ANR; the positive effect of reducing the correlation between carcass weight and leanness is surpassed by the negative effect of reducing the correlation between live weight (i.e. the trait on which the selection is based) and carcass weight (i.e. the trait on which the pig is price graded). Correlations (ρ_{CL,n,m}) between carcass leanness (CL) and other traits at different stages are the must difficult parameters to estimate, but reducing these correlations do not, even when reduced by 50%, affect the model output when selecting on observed live weight.
4.3 Results

4.3.2.2 Observational errors.

The results of conditioning the expected marginal net return \( (MNR_n) \) on observed live weight and given a high observational error (i.e. \( E_w = 8^2 \)) are shown graphically in fig. 4.10. Compared to fig. 4.6 the graphs of \( MNR \) are leveled out; the utility (or informational value) of observing the live weight is decreased and the expectations (i.e. the posterior distributions) of the traits are almost unaffected by the observations and are converging towards the prior expectations as the observational errors are increasing. As the average unconditioned \( MNR_n \) (i.e. based on the prior distribution) is higher at stage 1 than at stage 3 and as exemplified by the graphs, the difference between \( MNR_1 \) and \( MNR_3 \) is increasing as the observational error is increased.

The results of changing the observational error on live weight \( (E_w) \) are shown in table 4.8. For increasing values of \( E_w \), the optimal threshold value (i.e. \( w^* \)) is increased for \( n = 1, 2 \) replace and decreased for \( n = 3, 4 \). For \( E_w = 8^2 \) and for \( n = 1, 3, w^* \) in table 4.8 are seen to be the solutions

![Graph showing MNR and f(w*)](image)

**Figure 4.10.** Constant weaner supply. Marginal density of observed live weight \( (w^*) \) and expected marginal net return \( (MNR) \) conditioned on \( w^* \) at stage 1 and 3. \( E_w = 8^2, Z = 5,000 \).

**Table 4.8.** Constant weaner supply. Optimal live weight \( (w^*) \), average net return \( (ANR) \), optimal average net return \( (ANR^*) \) and optimal terminal marketing stage \( (n^*) \) given different levels of observation error \( (E_w) \) on live weight. OM = DO and SC = 11.

<table>
<thead>
<tr>
<th>( E_w )</th>
<th>( w^* ) (kg)</th>
<th>( ANR ) (DKK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_w )</td>
<td>( n - 1 )</td>
<td>( n - 2 )</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>94.1</td>
<td>93.7</td>
</tr>
<tr>
<td>( \frac{1}{4} )</td>
<td>93.9</td>
<td>93.6</td>
</tr>
<tr>
<td>( \frac{1}{8} )</td>
<td>93.9</td>
<td>93.6</td>
</tr>
<tr>
<td>( \frac{1}{16} )</td>
<td>93.9</td>
<td>93.6</td>
</tr>
<tr>
<td>( \frac{1}{32} )</td>
<td>94.1</td>
<td>93.7</td>
</tr>
<tr>
<td>( \frac{1}{64} )</td>
<td>94.7</td>
<td>93.8</td>
</tr>
<tr>
<td>( \frac{1}{128} )</td>
<td>97.1</td>
<td>94.4</td>
</tr>
<tr>
<td>( \frac{1}{256} )</td>
<td>107.2</td>
<td>98.0</td>
</tr>
<tr>
<td>( \frac{1}{512} )</td>
<td>115.6</td>
<td>89.8</td>
</tr>
</tbody>
</table>
Table 4.9. Constant weaner supply. Optimal terminal marketing stage \( (m^*) \) and optimal average net return \( (ANR^*) \) given different levels of observation error on live weight \( (E_w) \) and carcass leanness \( (E_{CL}) \). OM = DO and SC = III.

<table>
<thead>
<tr>
<th>( E_{w}^{1/2} )</th>
<th>( E_{CL}^{1/2} = 0.0001 )</th>
<th>( E_{CL}^{1/2} = 0.0004 )</th>
<th>( E_{CL}^{1/2} = 0.0016 )</th>
<th>( E_{CL}^{1/2} = 0.0064 )</th>
<th>( E_{CL}^{1/2} = 0.0256 )</th>
<th>( ANR^* ) (DKK)</th>
<th>( ANR^* ) (SC II)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1/2 )</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>6.469</td>
<td>6.469</td>
</tr>
<tr>
<td>( 1/4 )</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>6.468</td>
<td>6.468</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>6.451</td>
<td>6.451</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>6.357</td>
<td>6.357</td>
</tr>
<tr>
<td>16</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>6.186</td>
<td>6.186</td>
</tr>
</tbody>
</table>

to \( MNR_{w} \) \( (w^*) = 0 \) in fig. 4.10. The average net returns \( (ANR) \) are decreasing and converging toward the values of marketing without selection (i.e. SC I in table 4.3) as the observational error increases. In addition the optimal average net return is almost unchanged, when reducing the default value of the observational error (i.e. from \( E_{w} = 0.5^2 \)).

In table 4.9 the effects of changing the observational error on live weight \( (E_{w}) \) as well as on carcass leanness \( (E_{CL}) \) are shown and compared to the effect of changing the observational error on live weight only (i.e. as presented in table 4.8). The effects of changes in \( E_{CL} \) as well as the differences between SC II and III are (as expected) largest for large values of \( E_{w} \), but the magnitudes of the effects (and differences) are relative small. With a standard deviation of 16 kg on the observational error on live weight (i.e. \( E_{w} = 16^2 \)), the optimal average net return is decreased by approx. 0.016 DKK when \( E_{CL}^{1/2} \) is multiplied by 4\(^4 \) (i.e. changed from 0.0001 to 0.0256) and the difference between SC II and III is approximately 0.022 DKK per week per pig place (or 1.14 DKK per year per pig place) for \( E_{CL}^{1/2} < 0.01 \).

\[ \text{Figure 4.11. Random weaner supply. Optimal mean of weaner supply } (\mu^*) \text{ given different optimization methods (DO and PO) and expected number of excess weaners } (E \text{ (EP) and expected batch size } (E \text{ (BS)}), \text{ all given different standard deviations on weaner supply } (\sigma). K = 20, LEP = 50.00 DKK and SC = II. } E_{w} - 0.5^2. \]
4.3.2.3 Random weaner supply

In general some relationship between the standard deviation ($\sigma$) and the mean ($\mu$) of a random weaner supply may be expected (like the binomial distribution in which both the variance and the mean are linear in $n$, a parameter in the distribution), but in the following it is, without any further discussion, assumed that the mean value is changed independently of the standard deviation.

The results of optimizing $\mu$ given different levels of $\sigma$ and with a loss per excess weaner ($LEP$) of 50.00 DKK/weaner are shown in fig. 4.11 and 4.12. An increase in $\sigma$ leads to an increase in the expected number of excess weaners, $E(EP)$ and a decrease in the expected batch size, $E(BS)$ and eventually to a decrease in $ANR$. The difference between $DO$ and $PO$ is decreasing for $0 < \sigma < 5$ and increasing for $5 < \sigma < 26$, but the difference is below approx. 0.065 DKK for all $\sigma$. The optimal mean ($\mu^*$) and the difference between $DO$ and $PO$ is increasing as $\sigma$ is increased from 0 to approximately 20 weaners and decreasing for standard deviations above 20.

4.4 Discussion

When interpreting and discussing the results presented in the previous section it is import to recall that the slaughter pig marketing management is part of a larger and dynamically changing system. Therefore marketing management should, at least in principle, not be optimized/managed independently of the rest of the system. In addition the results are based on just a single finishing operation sample case and the parameters of the model are to some extend estimated from simulated data. Despite these provisos, the results might serve as indicators on how to manage marketing in general and the behavior of the presented model in particular.

The differences between weekly average net return per pig place ($ANR$) of different selection methods can be interpreted as the opportunity costs of applying one method instead of another. Assuming no other costs or risks implied by the methods and assuming that the terminal

![Graph](image)

Figure 4.12. Random weaner supply. Optimal average net return ($ANR^*$) given different optimization methods ($DO$ and $PO$) and different standard deviations on weaner supply ($\sigma$) and corresponding optimal mean on weaner supply ($\mu^*$) as shown in fig. 4.11. $K = 20$, $LEP = 50.00$ DKK and $SC = II$. $E_w = 0.5^2$. 

\[ ANR^*, PO \quad -- -- ANR^*, DO \quad -- -- Difference ANR^*, DO, PO \]
marketing stage is not subject to change, the value is directly comparable to the actual costs of applying the methods and the methods may be ranked based on this (a more correct or universal method of accessing the value of selection would be to include the cost of selection directly in the calculation of ANR). In the case of no selection at all (i.e. SC I) vs. selection on live weight (i.e. SC II), a cost of selection less than 0.30 DKK/week/pig place at stage 3 or approximately 4.50 DKK/pig would make selection profitable. At succeeding stages the difference is even larger; more pigs are selected and marketed at “the right time”. The profitability depends on the weighing precision, but only very high and unrealistic observational errors (i.e. $E_w > 1$) will significantly change the profitability and as also demonstrated by Jørgensen (1993) it may not even be beneficial to balance weight pigs prior to marketing compared to simple visual assessment.

Selection on live weight and carcass leanness (i.e. SC III) is (regardless of the observational errors on both traits) only slightly superior to SC II and very little financial room is left for measuring the leanness of pigs. However, alternative pricing systems, as well as including the additional benefit of utilizing the information in other management problems within the enterprise, can change this picture.

One question that may arise when optimizing and analyzing selections criteria is: What are the financial consequences of applying an in–optimal selection criterion? In fig. 4.13 the financial effect of choosing a wrong threshold value (which is applied at all stages, i.e. SC II) is shown. The loss (the difference between calculated optimal values as shown in table 4.4and the realized value given a specific threshold value) in ANR with a given threshold value appears to increase with the stage number; more pigs are selected as the termination of the batch is postponed. In the case of termination at stage 5 the loss of applying a threshold value of 96 kg (i.e. approximately 2.5 kg more than the optimal threshold value) is 0.30 DKK per pig place per week or approximately 5.10 DKK per produced pig. It should be noted how the using of same threshold weight at all stages in all situations (except terminal marketing at stage 2) lead to a loss.

The differences between perfect information and all other selection criteria (see e.g. table 4.4) indicate that it is possible to develop and apply better criteria, but still the profitability depends on the actual costs (and additional returns) of performing the measuring and selection.

![Figure 4.13](attachment:image.png)

**Figure 4.13.** Loss in ANR of using in–optimal threshold weights when selecting individuals. $Z = 400,000$
4.4 Discussion

The financial effect of changing the terminal marketing stage can be of considerable size, but in particular in batch operations the operational effects of changes can be very difficult to handle and fit into a tight schedule. Though, in some situations (and if the manager expects the current state of the herd to be permanent) the financial benefits are of such a magnitude that the operational problems caused by a change in the demand and supply of weaner are worthwhile. However, a simple change in feeding and growth of pigs (and consequently a change in demand of weaners) in the slaughter pig operation may in many situations be the optimal solution to disharmonies between weaner demand and supply.

The effect of changeable vs. unchangeable weaner supply is little and decreasing with increasing batch size and even in the situation of stochastic variation of the weaner supply the difference is little. Recognizing the inability of most systems to change weaner supply from week to week, it will in general serve no purpose to perform the calculations of probabilistic optimization (i.e. Dynamic Programming) instead of deterministic optimization.

In general all problems defined in the introduction can be handled by applying the model and methods proposed. The fundamental animal growth model is from a theoretical and comprehensible point of view and in relation to many other growth models (see e.g. Jørgensen, 1993; Whittemore, 1993; Chavas et al., 1985) quite simple; traits and growth are simply represented as (auto-)correlated stochastic variables. The use of growth functions is (in general) avoided, which may be of importance with respect to the users confidence in a future implementation as a Decision Support System. Despite – or maybe because of – the simplicity of the model, it appears to be very well suited for handling the problems of observational errors, of updating the expectation of pigs (in particular the latent traits as e.g. carcass weight and leanness) based on observations (i.e. Bayesian Updating as defined in (4.1)) and as a basis for the examination/optimization of the overall problems of when and how to select pigs and when (ideally) to terminate batches.

The major general disadvantage of this type of model is the limitations on size or degree of detail. The number of traits and stages has to be “small”. However, as the number of marketing stages normally is limited to less than 10 (i.e. 5 weeks duration of the marketing of a batch and maximum 2 marketings per week) this obstacle does not apply to the problem of slaughter pig marketing management and as discussed in chapter 2, 3 it is possible to generalize the animal growth model to a continuous model in which parameters are represented as functions of two points of time (e.g. of “marketing stage 2.6” and “stage 3.8”). Another problem (that would apply to any model of this management problem) is to assess model parameters. In particular the correlations between latent traits over time are difficult to estimate (i.e. these traits are only revealed once in the life time of a pig: at slaughter), but as indicated by the results, the model appears to be very robust with respect to errors in these estimates and as demonstrated in chapter 2, 3 it is possible to calculate good estimates based on realistic and sparse data sets.

Any reasonable selection criterion (based on traits included in the animal model) can be handled by the model and in principle the marginal net return of individuals (given any set of observations on the individual) can be calculated and used as a selection criterion. This selection criterion is not very operational when applied directly by the manager (an input and computing device is needed), but when combined with equipment for automated identification, feeding, weighing, backfat-measuring and separation of individuals the criterion will be operational as
well as beneficial.

The use of simulations for estimating integrals has resulted in a large quantity of statistics that are valuable in examining and validating the model, but superfluous when the model is applied to practical problems. These statistics may not easily be calculated when using traditional methods of numerical integration, but these numerical methods would be more efficient (from a numerical point of view) and should be applied when implementing the model as a decision support tool.

The presented marketing management model may be applied directly in order to calculate general or farm specific optimal threshold values as shown in table 4.2 and in order to calculate the optimal terminal marketing stage as an aid in managing the weaner demand and supply. As demonstrated by the results the influence of changes in model parameters on optimal threshold weights is little, implying that optimization of production independently of marketing management may be feasible and quite accurate. However, a more general utilization of the concept of the model directly in objective functions used for e.g. optimizing feed (energy) supply for pigs would result in a simultaneous optimization of production and marketing (as suggested by Chavas et al., 1985) and the optimal marketing decisions or strategy would then simply be a (sub–)result of the optimization. The same idea (i.e. of using the model as part of the objective function) may apply to the problem of (re–)dimensioning and planning the production facilities as well as other parts of managing the slaughter pig operation and the weaner supply/demand.

The proposed models and methods can be implemented as a DSS as they are, but a set of changes/enhancements would improve the value of such a system. The inclusion of seasonal variations and cyclic price variations are other relevant options, but other optimization methods (based on Dynamic programming) would have to be applied. The problem of how to market pigs as defined in the introduction appears, at least in some situations, to be the most important extension to add, and this is properly the area first in need of developing.
4.5 References


### Appendix 4A: Model parameters of the case finishing operation

#### Parameters of animal model: Means ($m$), variances and covariances ($C$) and correlations ($\rho$):

<table>
<thead>
<tr>
<th>$W_1$</th>
<th>81.7</th>
<th>$W_2$</th>
<th>87.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CW_1$</td>
<td>64.6</td>
<td>$CW_2$</td>
<td>69.0</td>
</tr>
<tr>
<td>$CL_1$</td>
<td>59.8</td>
<td>$CL_2$</td>
<td>59.4</td>
</tr>
<tr>
<td>$W_3$</td>
<td>92.9</td>
<td>$W_4$</td>
<td>98.3</td>
</tr>
<tr>
<td>$CW_3$</td>
<td>73.4</td>
<td>$CW_4$</td>
<td>77.7</td>
</tr>
<tr>
<td>$CL_3$</td>
<td>59.0</td>
<td>$W_5$</td>
<td>103.6</td>
</tr>
<tr>
<td>$CW_5$</td>
<td>81.9</td>
<td>$W_6$</td>
<td>81.9</td>
</tr>
</tbody>
</table>

#### $X$ - $m$ - $C$ - $\rho$

$$
\begin{bmatrix}
1 \\
0.971 & 1 \\
0.982 & 0.954 & -0.177 & 1 \\
0.954 & 0.983 & -0.197 & 0.972 & 1 \\
0.969 & 0.941 & -0.175 & 0.984 & 0.956 & -0.177 & 1 \\
0.941 & 0.969 & -0.194 & 0.956 & 0.984 & -0.197 & 0.973 & 1 \\
0.917 & -0.192 & 0.954 & -0.175 & -0.177 & 0.982 & -0.180 & -0.200 & 1 \\
0.957 & 0.929 & -0.172 & 0.971 & 0.944 & -0.175 & 0.985 & 0.957 & -0.177 & 1 \\
0.929 & 0.956 & -0.192 & 0.944 & 0.972 & -0.194 & 0.957 & 0.986 & -0.197 & 0.974 & 1 \\
0.917 & -0.192 & 0.954 & -0.175 & -0.177 & 0.982 & -0.180 & -0.200 & 1 \\
0.944 & 0.917 & -0.170 & 0.959 & 0.932 & -0.172 & 0.974 & 0.946 & -0.175 & 0.986 & 0.958 & -0.177 & 1 \\
0.917 & 0.945 & -0.189 & 0.932 & 0.960 & -0.192 & 0.946 & 0.974 & -0.194 & 0.958 & 0.987 & -0.197 & 0.975 & 1 \\
-0.170 & -0.189 & 0.941 & -0.172 & -0.192 & 0.954 & -0.175 & -0.194 & 0.968 & -0.177 & -0.197 & 0.982 & -0.180 & -0.200 & 1
\end{bmatrix}
$$
Other model parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Relation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>Interest rate on a weekly basis (5% pa.)</td>
<td>$i = 1.05^{1/2} - 1$</td>
<td>0.0938</td>
<td>%</td>
</tr>
<tr>
<td>$r$</td>
<td>Discounting factor</td>
<td>$r = 1/(1 + i)$</td>
<td>0.99906</td>
<td></td>
</tr>
<tr>
<td>$M$</td>
<td>Time from insertion to first marketing stage</td>
<td></td>
<td>12</td>
<td>weeks</td>
</tr>
<tr>
<td>EP</td>
<td>Empty period (time between batches)</td>
<td></td>
<td>1</td>
<td>weeks</td>
</tr>
<tr>
<td>$N$</td>
<td>The last terminal marketing stage of a batch</td>
<td></td>
<td>5</td>
<td>stage</td>
</tr>
<tr>
<td>WP</td>
<td>Weaner price</td>
<td></td>
<td>350.00</td>
<td>DKK/weaner</td>
</tr>
<tr>
<td>FP</td>
<td>Feed price</td>
<td></td>
<td>1.40</td>
<td>DKK/FU_p</td>
</tr>
<tr>
<td>FE</td>
<td>Feed efficiency prior to first marketing stage</td>
<td></td>
<td>2.75</td>
<td>FU_p/kg</td>
</tr>
<tr>
<td>DG</td>
<td>Daily gain (from insertion to first marketing stage; i.e. from 25 - 81 kg live weight)</td>
<td>$DG = \frac{81 - 25}{(M-EP)\cdot7}$</td>
<td>0.730</td>
<td>kg/day</td>
</tr>
<tr>
<td>PFC</td>
<td>Accumulated feed consumption prior to first marketing stage</td>
<td>$PFC = \frac{(M-EP)\cdot7\cdot DG \cdot FE}{(M-EP)\cdot7}$</td>
<td>154</td>
<td>FU_p</td>
</tr>
<tr>
<td>FPC</td>
<td>Fixed purchase costs</td>
<td></td>
<td>30</td>
<td>DKK/pig place</td>
</tr>
<tr>
<td>VPC</td>
<td>Variable purchase costs</td>
<td>$VPC = PFC \cdot FP + WP$</td>
<td></td>
<td>DKK/weaner</td>
</tr>
<tr>
<td>PC</td>
<td>Total purchase costs (if batch size = section size, i.e. if constant weaner supply)</td>
<td>$PC = PFC + VPC$</td>
<td></td>
<td>DKK/weaner</td>
</tr>
<tr>
<td>LEP</td>
<td>Loss of excess weaners</td>
<td></td>
<td>50</td>
<td>DKK/weaner</td>
</tr>
<tr>
<td>$k_1$</td>
<td>Gompertz growth curve. Energy consumption per kg live weight.</td>
<td></td>
<td></td>
<td>FU_p/kg</td>
</tr>
<tr>
<td>$k_2$</td>
<td>Energy consumption per kg live weight gain</td>
<td></td>
<td>5.30</td>
<td>FU_p/kg</td>
</tr>
<tr>
<td>$k_3$</td>
<td>Logarithm of outgrown weight.</td>
<td></td>
<td>0.011</td>
<td>ln (kg)</td>
</tr>
<tr>
<td>$k_4$</td>
<td>Growth rate parameter.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{\text{wr}}$</td>
<td>Variance of observational error on live weight.</td>
<td></td>
<td>0.5²</td>
<td>kg²</td>
</tr>
<tr>
<td>$E_{\text{cL}}$</td>
<td>Variance of observational error on carcass leaness.</td>
<td></td>
<td>0.002²</td>
<td>kg²</td>
</tr>
</tbody>
</table>
Recursive Dynamic Programming

Abstract. An alternative approach towards dynamic programming (DP) is presented: Recursions. A basic deterministic model and a solution function are derived from Bellman’s Principle of Optimality. The model is generalized to include stochastic processes, with the traditional stochastic DP model (or Markov decision programming problem) as a special case. Heuristic rules are included in the models simply as restrictions on decision spaces and it is demonstrated how the redundant problem representation and solving of the Value Iteration method is eliminated by use of this recursive approach.

Keywords. Dynamic programming, recursive algorithms, heuristics, stochastic modeling.

5.1 Introduction

Dynamic Programming (DP) may be characterized as a general optimization principle, which was first formulated by Bellman (1957). The principle is usually related to the field of Operations Research (OR), but the underlying and rather intuitive “Principle of optimality” (Bellman, 1957) is essential to several specific optimization techniques, including Hierarchic Markov Processes (Kristensen, 1988), A* (Winston, 1992) and the solution methods of Influence Diagrams and Valuation Networks (Shenoy, 1992).

Value Iteration (VI) is a well known and widely used DP technique. In this paper an alternative and more general approach towards DP is presented: Recursions. The approach (Recursive Dynamic Programming, RDP) is basically a direct implementation of the fundamental functional equation of DP (as defined by e.g. Nemhauser (1966)):
\[ f_n(x_n) = \min_{d_n \in D_n} \left[ r_n(x_n, d_n)^{x_n} f_{n-1}(t_n(x_n, d_n)) \right] \]  
(5.26)

where \( n \) is the stage number,

\( x_n \) is the state (\( n = 0 \) indicates a goal state),

\( f_n \) is the solution function at stage \( n \). Returns the solution to the problem associated with \( x_n \), \( f_n(x_n) \) is per definition a known value,

\( d_n \) is a decision, chosen from the set of valid decisions, \( D_n \),

\( r_n \) is the gain function and

\( t_n \) is the transition function.

The purpose of this paper is to show the simplicity, the generality and the potential computational benefits of RDP. It must be emphasized that this approach neither is nor intend to be novel, but it appears to be relatively unknown to OR and agricultural scientists (Kure, 1995).

The approach has, as will be shown, several theoretical and applicational implications. The methods presented may be applied to virtually any problem that can be solved using VI, including agricultural planning/optimization problems like facility management, replacement problems and farm operations scheduling. The methods (as defined in this paper) have been applied to simple illustrative problems and a larger slaughter pig marketing decision support system (see chapter 4).

All models and functions in this paper are specified and defined using the syntax of the specification language, VDM (see e.g. Jones (1990) for an introduction) which in many aspects is equivalent to functional languages as SML (see e.g. Paulson, 1991).

5.2 Recursive Dynamic Programming

An example will illustrate the model, the problem, the method and some implications of RDP: Consider a road map represented as a directed graph as shown in fig. 5.1. Each node (or state) in the graph represents a crossroads and each (directed) edge represents a piece of one-way road. A decision prescribes what edge to follow from a given node. Node A is defined as a start node. Node B has no successors; it is a goal node. Edges may be interpreted as a pair of functions:

a) A transformation function:
\[ t \colon \text{State} \times \text{Decision} \rightarrow \text{State} \]
returning a new node/state and

b) a gain function:
\[ g \colon \text{State} \times \text{Decision} \rightarrow \text{Gain} \]
returning a gain (i.e. the length of an edge).

A policy is (in this presentation) defined as an assignment of a (valid) decision to every node in a sub-set of nodes in the graph and a complete policy assigns a decision to every node (except
goal nodes) in the graph. The optimization problem to be considered here is: What is the shortest path from $A$ to $B$?

### 5.2.1 The deterministic data model

Based on these basic components, a deterministic data model/data type of a system is formulated as a finite mapping from states into a finite mapping from decisions into gains and (succeeding) states:

\[
DPM = \text{State} \sqsubseteq \text{Decision} \sqsubseteq \text{Gain} \times \text{State}
\]

\[
\text{inv} (dpm) \ldots
\]

where the invariant (inv) must restrict the model to be finite and cycle free (see Kure (1995) for a definition). The model will later be modified to be based on functions instead of finite mappings. In addition to the data model of the system a representation of a policy is required and defined as:

\[
\text{Policy} = \text{State} \sqsubseteq \text{Decision}
\]

### 5.2.2 The Problem

In order to formulate and solve the problem, a set of basic functions is specified and defined (see Jones (1990) for an introduction to specifications and direct definitions in VDM):

\[
\text{complete}_\text{pol} (dpm: DPM, p: \text{Policy}) : \text{boolean}
\]

\[
\text{post } r = (\text{dom } p = \text{dom } dpm \land \\
\forall \text{st} \in \text{dom } p : p (\text{st}) \in \text{dom } dpm (\text{st})
\]

\[
\text{all}_\text{pol} (dpm: \text{DPM}) : \text{Policy}-\text{set}
\]

\[
\text{post } r = \{p: \text{Policy} | \text{complete}_\text{pol} (dpm, p)\}
\]

\[
\text{combine}: \text{Gain} \times \text{Gain} \to \text{Gain}
\]

![Diagram](image)

**Figure 5.1.** A road map.
map: (X × Y, X-set) → Y-set

map (f, s) △
  if s = {} then {}
  else let e ∈ s in f(e) ∪ map (f, s-{e})

sub_dom: DModel → (State → State-set)

sub_dom (dpm)(st) △
  if st ∈ dom dpm then {}
  else let succ_states = map (snd, rng dpm (st))
        in succ_states ∪ map (sub_dom (dpm), succ_states)

all_pol returns all complete policies applicable to a given instance, dpm, of the model. sub_dom returns all succeeding states (the sub-domain) of a given state. The function combine (is defined later) is used in the definition of the objective function:

objfct: DModel × Policy × State → Gain

objfct (dpm, p, st) △
  let (g, n_st) = dpm (st)(p (st))
  in if n_st ∈ dom p then g
     else combine (g, objfct (dpm, p, n_st))

A minimization function is specified as:

min (s: Gain-set): Gain

pre s ≠ {}

post r ∈ s ∧ ∀ e ∈ s · e ≥ r

Given an instance of the model (dpm: DModel) and a state (st: State), the problem is to minimize the objective function (objfct) with respect to all complete policies (p ∈ all_pol (dpm)):

min { objfct (dpm, p, st) | p ∈ all_pol (dpm) }

In other words the goal is to find a solution function, solution, that satisfies the specification:

solution (dpm: DModel, st: State): Gain

pre st ∈ dom dpm

post r = min { objfct (dpm, p, st) | p ∈ all_pol (dpm) }

The state space is defined as the total set of states (except goal states) in the model and the decision space as the total set of valid decisions in a given state:
5.2 Recursive Dynamic Programming

\[
\text{state\_space (dpm) } \triangleq \text{ dom dpm}
\]
\[
\text{decision\_space (dpm, st) } \triangleq \text{ dom dpm (st)}
\]

### 5.2.3 Bellman’s Principle of Optimality

The derivation of a solution function will be based on Bellman's Principle of Optimality:

> The Principle of Optimality. An optimal policy has the property, that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

*Bellman (1957)*

Referring to the introduced data model, a proposition stating this principle may be formulated as:

**Proposition 1:**

\[
dpm \in \text{DModel}, \, st \in \text{State}, \, p^* \in \text{all\_pol (dpm)},
\]
\[
\min \{ \text{objfct (dpm, p, st)} \mid p \in \text{all\_pol (dpm)} \} = \text{objfct (dpm, p^*, st)}
\]
\[
\forall s \in \text{sub\_dom (dpm) (st)} \cdot \min \{ \text{objfct (dpm, p, s)} \mid p \in \text{all\_pol (dpm)} \} = \text{objfct (dpm, p^*, s)}
\]

which (because it is satisfied for $\forall st \in \text{dom dpm}$) implies:

**Proposition 2:**

\[
dpm \in \text{DModel}
\]
\[
\exists p^* \in \text{all\_pol (dpm)} \cdot \forall s \in \text{dom dpm} \cdot
\]
\[
\min \{ \text{objfct (dpm, p, s)} \mid p \in \text{all\_pol (dpm)} \} = \text{objfct (dpm, p^*, s)}
\]

### 5.2.4 solution – a direct derivation

Based on the introduced data model and functions and on the Principle of Optimality a direct definition of *solution* may be derived (and formally proved).

In the proof the following abbreviation will be used:

\[
f (d) = \text{let } (g, n_{\_st}) = \text{dpm (st)} (d)
\]
\[
\text{in } \begin{cases} n_{\_st} \in \text{dom dpm then } g \\
\text{else } \text{combine } (g, \text{objfct (dpm, p^*, n_{\_st}}))
\end{cases}
\]
Proof 1:

from \(dpm \in \text{DPModel}, \text{st} \in \text{dom} \ dpm\):
\[
\forall s \in \text{dom} \ dpm \cdot \text{solution} \ (dpm, s) = \min \{ \text{objfct} \ (dpm, p, s) \mid p \in \text{all}_\text{pol} \ (dpm) \};
\]

1. from \(p \in \text{all}_\text{pol} \ (dpm)\)
   infer \(p \ (\text{st}) \in \text{dom} \ dpm \ (\text{st})\)

2. from \(d^* \in \text{dom} \ dpm \ (\text{st})\); \(f \ (d^*) = \min \{ f \ (d) \mid d \in \text{dom} \ dpm \ (\text{st}) \}\);
   \(p^* \in \text{all}_\text{pol} \ (dpm)\); \(\forall s \in \text{dom} \ dpm \cdot \min \{ \text{objfct} \ (dpm, p, s) \mid p \in \text{all}_\text{pol} \ (dpm) \} = \text{objfct} \ (dpm, p^*, s)\)

2.2 \(p^* \{\text{st} \rightarrow d^*\} \in \text{all}_\text{pol} \ (dpm)\)

2.3 \(f \ (d^*) = \text{objfct} \ (dpm, p^* \{\text{st} \rightarrow d^*\}, \text{st})\)

2.4 \(\text{objfct} \ (dpm, p^* \{\text{st} \rightarrow d^*\}, \text{st}) \geq \text{objfct} \ (dpm, p^*, \text{st})\)

2.5 \(\text{objfct} \ (dpm, p^*, \text{st}) = f \ (p^* \ (\text{st}))\)

2.6 \(f \ (p^* \ (\text{st})) \geq f \ (d^*)\)

2.7 \(\text{objfct} \ (dpm, p^*, \text{st}) = f \ (d^*)\)

2.8 \(\min \{ \text{objfct} \ (dpm, p, \text{st}) \mid p \in \text{all}_\text{pol} \ (dpm) \} = \min \{ f \ (d) \mid d \in \text{dom} \ dpm \ (\text{st}) \}\)

infer \(\min \{ \text{objfct} \ (dpm, p, \text{st}) \mid p \in \text{all}_\text{pol} \ (dpm) \} = f, h2\)

min \{ let \(g, n_{\text{st}} = dpm \ (\text{st})(d)\)
   in \ if \(n_{\text{st}} \in \text{dom} \ dpm \) then \(g\)
   else combine \((g, \min \{ \text{objfct} \ (dpm, p, n_{\text{st}}) \mid p \in \text{all}_\text{pol} \ (dpm) \})\)
   | \(d \in \text{dom} \ dpm \ (\text{st})\}\)

3. \(\exists d^* \in \text{dom} \ dpm \ (\text{st}) \cdot f \ (d^*) = \min \{ f \ (d) \mid d \in \text{dom} \ dpm \ (\text{st})\}\)

4. \(\exists p^* \in \text{all}_\text{pol} \ (dpm) \cdot \forall s \in \text{dom} \ dpm \cdot \min \{ \text{objfct} \ (dpm, p, s) \mid p \in \text{all}_\text{pol} \ (dpm) \} = \text{objfct} \ (dpm, p^*, s)\)

infer \(\text{solution} \ (dpm, \text{st}) = \exists \cdot \text{E}, h, h\)

min \{ let \((g, n_{\text{st}}) = dpm \ (\text{st})(d)\)
   in \ if \(n_{\text{st}} \in \text{dom} \ dpm \) then \(g\)
   else combine \((g, \text{solution} \ (dpm, n_{\text{st}}))\)
   | \(d \in \text{dom} \ dpm \ (\text{st})\}\)
By introducing two new functions an alternative solution function that is closer to a direct implementation may be derived:

\[
\begin{align*}
\text{min\_two: } & \quad X \times Y \rightarrow Y \\
\text{min\_two (a, b)} & \triangleq \text{if } a \triangleq b \text{ then } a \text{ else } b \\
\text{minimum: } & \quad (X \rightarrow Y) \times X \rightarrow Y \\
\text{minimum (g, as)} & \triangleq \\
& \quad \text{let } \_\text{pre} (m, \_\text{as}) = \\
& \quad \text{if } \_\text{as} = \{\} \text{ then } m \text{ else } \\
& \quad \text{let } a = \_\text{as} \\
& \quad \text{in } \_\text{pre} (\text{min\_two (m, g (a)), \_as -} \{a\}) \\
& \quad \text{and } \_\text{as} \leftarrow a \\
& \quad \text{in } \_\text{pre} (g (\_\text{as}), \_\text{as} \leftarrow \{\_\text{as}\}) \\
& \quad \text{pre } \_\text{as} \leftarrow \{\}
\end{align*}
\]

The definition of the infix operator \(\triangleq\) will depend on the actual definition of Gain (see below).

Utilizing that:

\[
f : X \rightarrow Y, Y \rightarrow \{\} = \min \{f(x) \mid x \in X\} = \text{minimum (f, X)}
\]

solution can be deduced (by \(-\text{subst}\)) to:

\[
solution \triangleq \text{DPModel} \times \text{State} \rightarrow \text{Gain} \\
solution (\text{dpm}, \text{st}) \triangleq \\
& \quad \text{let } f (d) = \\
& \quad \quad \text{let } (g, \_\text{st}) = \text{dpm (st)}(d) \\
& \quad \quad \text{in } \text{if } \_\text{st} \in \text{dom dpm then } g \\
& \quad \quad \text{else } \text{combine (g, solution (dpm, \_\text{st}))} \\
& \quad \quad \text{in } \text{minimum (f, dom dpm (st))} \\
& \quad \quad \text{pre } \_\text{st} \in \text{dom dpm}
\]

The function minimum minimizes the returns of the function \(g\) over \(\_\text{as}\), a finite set of arguments to \(g\). In solution, minimum is utilized to minimize \(f\) over the finite set of valid decisions, \text{dom dpm} (\text{st}), where \(f\) returns the optimal gain of executing a decision, \(d\). The problem of finding the shortest path from A to B in fig. 5.1 is solved recursively starting in the start state (A) using solution.

### 5.2.5 Memory functions

As shown in the example, RDP consists of a recursively decomposition of a problem into and solving of sub–problems, but the solution function, solution, does not prevent the same problem
to be solved more than once (i.e. in the example different paths may lead to the same node). This (possibly inefficient) resolving of the same problem may be eliminated by keeping records of known solutions: If a solution is in the records, it is already known and may be used directly; Otherwise the problem must be solved and the result saved in the records for later use. This use of record keeping or use of a memory function (Brassard and Bratley, 1988) is an (or the) essential part of DP and may result in great savings in computing time.

The records of solutions can be represented as a map:

\[ \text{Solutions} = \text{State} \rightarrow \text{Gain} \]

If knowledge of the policy leading to the solution is required, this optimal policy may be represented as an extension to the records of solutions:

\[ \text{SolutionsOptPolicy} = \text{State} \rightarrow \text{Gain} \times \text{Decision} \]

All solution functions in this paper can be extended with memory functions (see Kure (1995) for direct definitions).

### 5.2.6 Function based model

The basic data model may be modified to be based on functions instead of (finite) maps:

\[ \text{NotInDomain} = \text{State} \rightarrow \text{boolean} \]
\[ \text{DecisionSpace} = \text{State} \rightarrow \text{Decision} - \text{set} \]
\[ \text{Consq} = \text{State} \times \text{Decision} \rightarrow \text{State} \times \text{Gain} \]

\[ \text{DPModelFunc} = \]
\[ \text{NotInDomain} \times \text{DecisionSpace} \times \text{Consq} \]
\[ \text{inv} \,(\text{dpm}) \triangleq \ldots \]

where \text{NotInDomain} restricts the domain of the model (or the problem space), \text{DecisionSpace} returns the decision space of a given state and \text{Consq} returns the consequences (i.e. gain and succeeding state) of executing a decision.

Based on this data model the basic solution function may be changed into:
5.3 Generalized RDP

\[ \text{solution\_func: } \text{DPModelFunc} \times \text{State} \to \text{Gain} \]

\[
\text{solution\_func (dpm, st)} \triangleq \\
\begin{cases}
\text{let } (\text{not\_in\_dom, decs, consq}) = \text{dpm} \\
\text{and } f (d) = \\
\text{let } (g, n\_st) = \text{consq (st, d)} \\
\text{in if } \text{not\_in\_dom (n\_st)} \text{ then } g \\
\text{else } \text{combine (g, solution\_func (dpm, n\_st))} \\
\text{in } \text{minimum (f, decs (st))}
\end{cases}
\]

The flexibility of this model and solution function will be exemplified in section 4.

Selected parts of a SML–implementation (see Paulson (1991) for an introduction) of this algorithm is shown in Appendix A. The function not\_in\_dom (of type: NotInDomain) is in this implementation a global function and not a part of the data model.

5.3 Generalized RDP

The basic deterministic model, DPModel can be generalized to include stochastic processes that satisfy the Markovian property (Hillier & Lieberman, 1986):

A stochastic process \( \{X_t\} \) is said to have the Markovian property if:

\[
P (X_{t+1} = j | X_0 = k_0, X_1 = k_1, \ldots, X_{t-1} = k_{t-1}, X_t = \ell) = P (X_{t+1} = j | X_t = \ell)
\]

for \( t = 0, 1, \ldots \) and every sequence \( i, j, k_0, k_1, \ldots, k_{t-1} \).

The equivalence (i.e. “the independence between the present state of a process and past events”) between this property and Bellman’s Principle of Optimality is utilized as follows.

A general summation function used in the definitions is defined as:

\[ \text{sum: } (X \to Y) \times X\text{-set} \to Y \]

\[
\text{sum (f, as)} \triangleq \\
\begin{cases}
\text{let } L\_sum (s, L\_as) = \\
\text{if } L\_as = \{\} \text{ then } s \\
\text{else let } a \in L\_as \\
\text{in } L\_sum (s \circ f (a), L\_as - \{a\})
\end{cases}
\]

and \( \text{fst}_a \in \text{as} \)

\[
\text{in } L\_sum (f (\text{fst}_a), \text{as} - \{\text{fst}_a\})
\]

pre as \( \neq \{\} \)

The infix operator \( \circ \) will be defined later.
5.3.1 The model

Let stochastic events (or states) be represented by the data type ChanceState. The Generalized RDP model is then defined as:

\[ \text{Prob} = \{ x \in \mathbb{R} \mid 0 \leq x \leq 1 \} \]

\[ \text{GenState} = \text{State} \cup \text{ChanceState} \]

\[ \text{DecConsq} = (\text{Decision} \times \text{Gain} \times \text{GenState})-\text{set} \]

\[ \text{ChanceConsq} = (\text{Prob} \times \text{Gain} \times \text{GenState})-\text{set} \]

\[ \text{inv} (c) \triangleq \text{sum} (\text{fst}, c) = 1 \]

\[ \text{GenConsq} = \text{GenState} \rightarrow (\text{DecConsq} \cup \text{ChanceConsq}) \]

\[ \text{inv} (gc) \triangleq \ldots \]

\[ \text{GDPModel} = \]

\[ \text{NotInDomain} \times \text{GenConsq} \]

\[ \text{inv} (dpm) \triangleq \ldots \]

It should be noted how DecisionSpace and Consq have been collapsed into DecConsq (and GenConsq).

The graph in fig. 5.2 is an example of an instance of this model and it illustrates an important feature of the model: Decision nodes and chance nodes can appear in any order.

Stochastic DP models (see e.g. Nemhauser, 1966) and Markov Decision Processes (see e.g. Kristensen, 1988) are seen to be special cases of GDPModel, in which a State always (unless it is a goal state) is succeeded by ChanceStates and vice versa. In those special cases the representation of chance states is superfluous and the model can be reformulated as:

![Diagram of a General DP model](image)

**Figure 5.2.** An instance of a General DP model. Decision nodes may be succeeded by decision nodes as well as chance nodes (and vice versa). Nodes represent values (and not variables as in Bayesian Networks).
StochConsq =
\[ State \rightarrow (Decision \times (Prob \times Gain \times State)\text{-set})\text{-set} \]

SDPModel =
\[ \text{NotInDomain} \times \text{StochConsq} \]
\[ \text{inv (sdpm)} \triangleq \ldots \]

5.3.2 The solution function

The solution function (without memory functions; see Kure (1995) for definitions of solution functions with memory functions) for the Generalized RDP model is defined as:

\[
\text{solution\_gen}: \text{GDPMo}\text{del} \times \text{GenState} \rightarrow \text{Gain}
\]
\[
\text{solution\_gen} ((\text{not\_mem}, \text{consq}), \text{st})\triangleq
\]
\[
\text{if } \text{st} \in \text{State}
\]
\[
\text{then let } f ((d, r, n\_st)) =
\]
\[
\text{if } \text{not\_mem} (n\_st) \text{ then } r
\]
\[
\text{else } r \circ \text{solution\_gen} ((\text{not\_mem}, \text{consq}), n\_st)
\]
\[
\text{in } \text{minimum} (f, \text{consq} (st))
\]
\[
\text{else let } f ((p, r, n\_st)) =
\]
\[
\text{if } \text{not\_mem} (n\_st) \text{ then } p \circ r
\]
\[
\text{else } p \circ (r \circ \text{solution\_gen} ((\text{not\_mem}, \text{consq}), n\_st))
\]
\[
\text{in } \text{sum} (f, \text{consq} (st))
\]

which is seen to be an extended version of \text{solution\_func}: if the state \text{st} is a \text{ChanceState} the weighted (i.e. by the factor or probability, \text{p}) gains are summed up; if not, the gains are (exactly as in \text{solution\_func}) minimized. In order to clarify the definition, the function \text{combine} has been substituted by the infix operator \( \circ \) (i.e. \text{comb} (a, b) \triangleq a \circ b)

It should be noted that the decisions, \text{d}, are not used in the calculations and may as well be removed from the data model, \text{DecConsq}. This demonstrates the fact that from a conceptually point of view the decisions (and policies) only serve the purpose of “naming” edges (and paths). However, if memory functions are used (which will normally be the case) it may be of relevance to record this information of the “name” of the optimal path together with the solution (the optimal gain).

5.3.3 Definitions of the type \text{Gain} and of the operators

In the most basic situation the gain and the operators used in the minimization and solution functions are defined as simple addition and minimum operations:

\[
\text{Gain} = \mathbb{R}
\]
\[
\circ : \text{Gain} \times \text{Gain} \rightarrow \text{Gain}
\]
\[
\circ (a, b) \triangleq a + b
\]
\( \odot : \text{Prob} \times \text{Gain} \rightarrow \text{Gain} \)
\( \odot (p, b) \triangleq p \cdot b \)
\( \odot : \text{Gain} \times \text{Gain} \rightarrow \text{Gain} \)
\( \odot (a, b) \triangleq \begin{cases} a & \text{if } a < b \\ b & \text{else} \end{cases} \)

In the situation of discounted returns, the gain and the operators are redefined as:

\[
\text{Gain} = (\mathbb{R} \times \mathbb{R})
\]
\( \odot ((g, q), (\text{sol}_g, \text{sol}_q)) \triangleq (q \cdot (g + \text{sol}_g), \text{sol}_q \cdot q) \)
\( \odot (p, (r, q)) \triangleq (p \cdot r, p \cdot q) \)
\( \odot ((r1, q1),(r2, q2)) \triangleq \begin{cases} (r1, q1) & \text{if } r1 < r2 \\ (r2, q2) & \text{else} \end{cases} \)

In this case the gain function defined in the introduction for each state (and each decision) returns a gain as well as a corresponding discount factor and the solution functions return (in addition to the discounted gain) the total discount factor given the optimal policy in the case of a deterministic model (i.e. solution) and the weighted total discount factor given the optimal policy in the case of a general model (i.e. solution_gen).

5.4 Implications

5.4.1 Heuristic rules

Without significantly affecting the optimal solution a heuristic rule for each state in the state space reduces the decision space and consequently the state space itself. These rules (and restrictions on decision spaces in general) are simply included in the models as modifications of DecisionSpace.

A road map (continued). The example will illustrate how restrictions on decision spaces can reduce the state space. The rule/restriction applied to the example is as follows: "Always follow the road at the left if you have the option." Fig.5. 3 illustrates this situation; roads "at the right" have been erased from the original graph (fig. 5.1) and only one path from A to B remains. The state space has been reduced to 4 states (A, N1, N3 and N5) and the recursive solution function is "reduced" to a simple summation function; \( \text{min}_\text{two} \) is never called.

5.4.2 Bounding

Heuristic functions (which return an under (over) estimate of the solution to a minimization (maximization) problem, see e.g. Rich and Knight, 1991) may without effecting the solution to the problem reduce the problem space. The problem space is reduced by bounding sub–graphs which with certainty will not lead to a better possible global solution than the current best known possible solution.

Kure (1995) defines a solution function for the basic model (RDPModel) which utilizes heuristic functions. The function is a special case of an algorithm known in AI as A* (Rich & Knight, 1991; Winston, 1992): The search strategy is a depth first search instead of an "intelligent" mixture of breadth first and depth first search as in A*.
A similar solution function can be defined for the general DP model and based on that function an A* algorithm for stochastic problems can be derived.

5.4.3 Value Iteration

In the original solution method proposed by Bellman (1957) and termed Value Iteration (VI) by Howard (1960) (or backward multistage problem solving by Nemhauser (1966)) the problem space is partitioned into stages. Each stage contains problems that can be solved independently and in any order. The stages are solved in an order which guarantees that all sub–problems (i.e. problems associated with succeeding states) already have been solved and recorded (problems associated with goal states are initially known). Fig. 5.3 illustrates one valid partitioning of the graph of fig. 5.1 (with heuristic rules as presented above); problems in stage 1 are solved first, followed by problems in stage 2, etc.

Fig. 5.3 also illustrates one of the main disadvantages of VI: Redundant problem solving. Problems (like those associated with node N2, N4, N6 and N7) which are not on a path leading from the start node will be solved, but they will never be reused. In contrast to RDP (where only problems on paths leading from the start node are represented and solved) there is in general no easy way (except processing the reduced state space prior to the optimization) to avoid this redundant problem solving in the VI algorithm.

The memory usage due to the recursive nature of the algorithm is the main disadvantage of RDP compared to VI. However, the memory complexity (or the memory usage) is linear in the depth of the problem or the number of stages and will therefore rarely cause any significant problems. It should be noted that in the situation of (infinite and) stationary problems, special procedures for stopping the recursions and for ensuring that all states are traversed at least once has to be applied when using RDP.

5.4.4 Hierarchic Markov Processes

It was demonstrated how the models and solution methods can be generalized to include stochastic (Markov) processes, with the Markov Decision Programming problem as a special case. Kristensen (1988) proposed the Hierarchic Markov Process (HMP) approach to solve MDP problems; a method partly based on the VI method, partly on the Policy Iteration method proposed by Howard (1960). In the HMP algorithm the weighted total discount factor is calculated explicitly while it in the Generalized RDP method (i.e. Combining with discounting.

![Figure 5.3. The graph of fig. 5.1 partitioned into stages and applied the rule: "always follow the road at the left if you have the option."](image-url)
above) is implicitly given; the state space is traversed once and not twice as in HMP. Combined
with the ability to include heuristic rules and in general avoid the representation and optimization
of states which are not on a path from the start state(s) the RDP method has the potential of
reducing the computational burdens of the HMP method.

5.5 Conclusion

Several aspects of DP have been examined and discussed in this paper. Some of these need
special considerations.

The basic solution function, solution described in section 2 is seen to simply be a
reformulation (with combine substituted by the infix operator ‘+’ and the stage number interpreted
as a state variable) of (5.1). Despite this very close and obvious relationship, the recursive
approach towards DP presented in this paper, may seem new to most people dealing with DP and
it appears difficult to find the approach described in literature (Kure, 1995). It is demonstrated
how the function can be directly derived based on a mathematical formulation of Bellman’s
Principle of Optimality – an approach that may aid in clarifying, interpreting and understanding
the principle.

The recursive approach implies easy implementation of heuristic rules (and of implicit and
explicit state space and decision space restricting rules in general). This feature may appear very
beneficial in practical applications: Any appropriate rule is easily applied to the model and only
states which according to the rules are on a path from the start state will be represented and
solved. This easy application of heuristic rules and the elimination of redundant problem solving
(as e.g. discussed by Houben et al. (1994)) is from an applicational point of view the most
interesting feature. From a theoretical point of view the flexibility, generality and implications
of the models and solution functions are of greater interest.
5.6 References


Appendix 5.A.   A SML—Implementation

Fragments of program:

...  
  type State = Stage * StateVar
  type DPModel = State -> (Decision list * (Decision -> (Gain * State)));
  ...
  fun minimum (f, []) = raise Error
  | minimum (f, fst_d::d_set) =
    let
      fun l_min (m, []) = m
      | l_min (m, d::l_d_set) =
        l_min (min_of_two (m, f(d)), l_d_set)
    in
      l_min (f(fst_d), d_set)
  end;

  fun solution (dpm, st) =
    let
      fun f(d) =
        let
          val (g, n_st) = snd(dpm(st))(d)
        in
          if n_st not_in_dom dpm
            then g
            else combine (g, solution (dpm, n_st))
        end
      in
        minimum (f, fst(dpm(st)))
    end;

Corresponding fragments of output:

...  
  type State = Stage * StateVar
  type DPModel = State -> Decision list * (Decision -> (Gain * State))
  ...
  val minimum = fn : ('a -> Gain) * 'a list -> Gain
  val solution = fn : (int * 'a -> 'b list *
                       ('b -> real * (int * 'a))) * (int * 'a) -> real
General discussion

As stated in the general introduction the overall objective of the research project was to investigate and develop general models for solving the “slaughter pig marketing management problem” utilizing herd specific registrations and with the aim of implementation as a slaughter pig marketing management support tool (MMST) to be used in the individual slaughter pig operation. In chapter 2 it was demonstrated how such a tool can be represented by two separated systems; a Belief Management System used for dynamic estimation of the (general – i.e. not just marketing management specific) “belief in the state of the herd”, and a Decision Support System, which based on the current “belief in the state of the herd”, returns information that may support the manager in managing the marketing of slaughter pigs. Fundamental and general models and methods for these sub–systems were individually described, evaluated and discussed in chapter 4 and 5 respectively. It was, at least from a theoretical point of view, demonstrated how the models can be applied in order to handle and solve “the slaughter pig marketing management problem” (i.e. how to select and when to market individual pigs or groups of pigs), given the special features of the slaughter pig operations (biological variation, operational constraints and evaluation/pricing systems; see chapter 1, the General Introduction) and given different levels of data/information (in the individual slaughter pig operation).

In this concluding chapter the main limitations, implications, perspectives and conclusions of the research will be outlined and aspects of implementing the models/methods as an operational and beneficial MMST will be discussed.

6.1 General limitations

The assumption of identically normally distributed traits of pigs of same age (and gender) is one of the main assumptions in the proposed models and methods and the assumption causes theoretical and applicational limitations. As discussed in the introduction, the manager will in most situations try to reduce the variation between marketed pigs in order to increase the financial return. A simple, useful and commonly applied method is to expedite or postpone the marketing of individual pig – or in other words to select pigs for marketing. As demonstrated in the
preceding chapters the proposed models/methods are capable of handling this situation, but if the reductions in variances (and the disturbances in distributions) are caused by other elements, like e.g. discrimination on feeding or selection of (best) individuals for breeding purposes, other models/methods have to be applied. The optimization methods do not directly rely on the assumption of normally distributed traits and can be extended to handle other distributions, while the parameter estimation methods (of the BMS) do rely on the normality assumption and are not easily extended. However, the contribution to reduction in variances from these elements are in most situations neglectable (slaughter pigs of same gender and age/batch are rarely discriminated on feeding and only very few, if any, pigs are selected from slaughter pig operations for breeding purposes) and the general gain of applying such revised and more complicated models/methods is expected to be very little.

The other two main assumptions – that (i) the remaining pigs in a pen/batch are unaffected by the selection of individual pigs for marketing and (ii) the age or (batch–#) of the individual pig is actually known and registered at slaughter – do also influence the feasibility and applicability of the models/methods. Assumption (ii) is in most slaughter pig operations not satisfied; the batch–# of pigs are normally not known at the time of slaughter. However, there are or should (at least under Danish conditions) be no significant practical or financial obstacles in getting this information of the batch–#. If pigs are individually identified and the identification mapped from farm to packer, the assumption will be satisfied.

6.2 The animal growth model

The animal growth model adopted in this study is from a mathematical point of view and compared to the traditional mechanistic and empirical models (see chapter 3) quite simple. However, the model appears to be very appropriate as a general model of the growth in the slaughter pig operation at the time of marketing and as an interface between the Belief Management System (BMS) and the marketing management Decision Support System (DSS).

As demonstrated and discussed in chapter 3 the model may be generalized to continuous time (as contrary to the discrete 5–stage models as used in this thesis). It was also discussed how the model may serve as an extension to traditional explanatory mechanistic models; an extension in which the mechanistic models are extended with a stochastic element (i.e. variation) and in which the mechanistic models are calibrated based on herd–specific registrations, using the basic idea of adding empirical Bayesian procedures e.g. as described by Oltjen (1992).

The use of the EM–algorithm to estimate and the Kalman–filter to update parameters in the animal growth model has shown that good and up–to–date estimates of mean (and variance) of the animal growth model may be derived from sparse farm–specific data–sets. The expected increase in computing power in the future, will enhance the relevance and applicability of these methods.

6.3 Optimization methods

As demonstrated in chapter 4, the problem of optimizing the marketing of slaughter pig may be (and is most appropriately) partitioned into two problems, that can be solved independently. However, very few of the reviewed studies on slaughter pig marketing optimization (Rasmussen,
1976 is one of the few exceptions) examine and discuss this aspect of marketing management and in this kind of studies there exist a general tendency to ignore the operational constraints that apply to a slaughter pig operation.

The dependency between the marketing management and other management tasks within the operation is another aspect that is generally ignored. Such aspects as feed diets and marketing management are in most situations (in theoretical studies as well as in practical applications) handled and optimized independently; However, they are not independent and should be considered jointly. The models and methods proposed in this study can be used for this purpose, f. ex. by extending traditional animal growth models with a stochastic element as discussed above or as a tool for analyzing the effect of changes in feed diets on the performance of the herd and the marketing of pigs – a tool that could be incorporated in traditional deterministic feed diet optimization programs.

The models and methods might be extended to handle and solve not only the problem of when to market pigs, but also the problem of when and how to market pigs (see the General Introduction for a discussion of these two aspects). This is (at least at present) not of relevance in the Danish market, but it is in other markets and the problem might constitute an interesting new/extended area of research.

### 6.4 Recursive Dynamic Programming

The recursive approach to Dynamic Programming (DP) presented in this thesis was a spin-off of the initial part of the study: DP was expected to play an important part of the study and was consequently examined thoroughly. This examination was based on an alternative approach – recursions – which revealed new or disregarded aspects of DP as demonstrated and discussed in chapter 5. The approach may assist newcomers and others in understanding the basically simple, but to many, very incomprehensible, *Principle of Optimality* and the underlying idea of DP; the confusing concepts of “stages” and of “in which order to solve stages” are avoided and substituted by a more direct and intuitive approach: *Recursions*. From a theoretical point of view, the method is both simple, flexible and from a computational point of view efficient and in addition very general compared to the traditional approach: *Value Iteration (VI) or Backwards multistage solving*. Decision and chance processes might be mixed in any amount and order (see chapter 4 for applied examples of this aspect) and without any unnecessary computations. This is in contrast to the traditional implementation of VI in which new “artificial dummy states” have to be included (and calculated) in order to handle such unorganized patterns of processes.

### 6.5 Implementation as a Marketing Management Support Tool

As discussed in the general introduction (chapter 1 of this thesis) the initiating idea or overall aim of the study was to develop a tool (i.e. a Marketing Management Support Tool, MMST) that based on herd-specific registrations would assist the manager in managing the marketing of slaughter pigs from the individual herd. It was discussed how to make such a system beneficial to the user and the enterprise owner: Good information. In the following the desired and expected output of a MMST based on the models and methods proposed in this thesis will be outlined and it will be discussed how to assess the expected value of using such a system.
Instructions on how, in the most profitable manner, to select individual pigs or groups of pigs for marketing should be considered as the main output of the MMST. Optimal threshold weights are examples of such instructions, but optimal combinations of live weight and leanness may be relevant and applicable as well. As demonstrated in chapter 4 (and primarily because of the shape of the price function, see e.g. figure 1 in chapter 4) the “optimal threshold weights” appear to be quite insensitive to changes in model parameters; the growth rate or the change in weight over time is of less importance than the absolute weight, when optimizing threshold weights. Consequently, the variation in optimal threshold weights between operations will be of a little magnitude and it might be argued that the managers do not need such a tool – all they need are general guidelines as discussed below. However, there are differences in growth and the general performance of pigs between operations and as demonstrated below there might be other benefits of using such tools, benefits that solely can justify the use of the tools.

Optimal terminal marketing stages are another important output of the MMST. Most confinement based operations are quite inflexible with respect to changing schedules and terminal marketing stage. However, in certain situations it is relevant and beneficial to change the terminal stage (e.g. after transition to a higher health status or another breeding line) and information on the effect of such changes can be very valuable, e.g in estimating the missed return of not changing the terminal stage. In less constrained operations (e.g. outdoor production) the tool will be even more relevant.

The tool returns other valuable information, than just information on optimal marketing. The marketing management optimizations will as a spin–off return estimates of future marketings of pigs; the number and distribution of pigs will be known several weeks in advance of the actual marketing and the tool might assist in scheduling the actual marketings. Another spin–off is the value of dynamic estimates of the “state of the herd”. These estimates can be used in a general monitoring of the herd; significant changes in the performance of the herd are easily and quickly detected. In addition the manager can use the tool for analyzing the effect of changes in model parameters (like prices) or calculate the consequences of performing in–optimal marketing; “What are the operational and financial consequences of omitting a marketing from a batch or of not selecting any individual pigs at all?”.

It seems reasonable to assume that the quality and quantity of available herd and animal specific registrations will increase in the future as new techniques and technologies and methods for cheap and reliable identification of individuals evolve. The computing power on the individual slaughter pig operation will continue to increase in the future and consequently the relevance of the methods and the value of using a MMST will increase. It might even be so that, some day the growth rates etc. of individual pigs are continuously estimated (e.g. once a day ) and feeding and marketing of the individuals are managed jointly and continuously based on this knowledge.

Estimating the value of using a tool like the one described above is a difficult task. Verstegen et al. (1995) distinguishes between two main evaluation approaches: The normative and the positive approach. In the normative approach the expected benefits of decision improvements resulting from the information supplied from the tool are aggregated in order to estimate the total benefit of using the tool. In most situations these benefits are very difficult to predict and assess and it could turn out to be difficult to separate the (in some situations very inconsistent) decisions and determine which information was used for a specific decision. In addition the benefits are more or less tangible: not all benefits are expressed in monetary value and the benefits may be difficult
to aggregate into a common denominator (e.g. utility). The normative approach is best suited for tools supporting very specific and well-defined decisions. To some extend this applies to the slaughter pig marketing management problem, but several of the benefits (e.g. the benefit of monitoring the herd) discussed above will be very difficult to estimate or assess. However, it is probably possible to estimate the effect of applying individualized selection criteria (i.e. threshold weights) in the individual operation, by optimizing different levels of growth rate (and feed conversion rate), calculating loses given a general guideline and combine these loses with the actual distribution of slaughter pig operations.

In the positive approach the tool is applied in a more or less experimental setting and the effect of the tool on decisions and (financial) return is observed over a given period of time (usually several years). The approach put high demands on the availability and quality of field data and the type of research design and cannot be used to estimate the expected value of a MMST prior to actually applying the tools.

A third approach which was not discussed by Verstegen et al. (1995) is to simply ask the potential users of the tool: “This is our product. Do you want it and what are you ready to pay for it?”. This approach is very simple and applicable, but indeed is does not guarantee that the output of the questions is consistent with the actual value of the tool. However, the skills and judgement of the managers should not be underestimated and the approach would probably, if combined with a normative evaluation as described above, be the most appropriate in the case of a Marketing Management Support Tool.

6.6 Other applicable issues

The variation in optimal threshold weights between operations may (as discussed above) be of a little magnitude and consequently general guidelines (as applied today, at least in Denmark) very applicable and beneficial within the individual operation. A wrong threshold value, however could and as demonstrated in chapter 4 be costly to the farmers and guidelines should be accurate and up-to-date. The models and methods proposed in this thesis are useful for this purpose, f. ex. with guidelines presented in a matrix format with optimal threshold values for different levels of mean growth rate and mean leanness and with a sensitivity analysis (like figure 6 in chapter 4) of expected costs of missing the target: “the optimal weight”.

The designing (e.g. the size and number of sections in a confinement) of new and the redesigning of old facilities are based on calculations of expected flows (duration and size) of pigs or batches of pigs. These calculations are normally based on expectations (on primarily the growth of pigs and duration of batches), ignoring the effect of variation on important variables (and parameters like prices): Duration of a batch (in weeks) = (exp. age at insertion − exp. age at the “optimal threshold weight”)/7 + 1. This way, half of the batch is expected to be marketed prior to the termination, but as demonstrated in chapter 4 the fraction of pigs marketed prior to termination might vary considerably (from 18.9% in the case of a 10% increase in price to 50.8% in the case of a 10% decrease in price). The methods proposed can be used to calculate the optimal termination stage (and the financial and operational consequences of in–optimal termination stages) – i.e. the “true optimal duration of batches” – as an aid in designing/dimensioning production facilities.

The proposed models and methods are intended for use in the individual slaughter pig operation, but the methods may as well be applied in the opposite end of the chain: At the
slaughter house/packer. The packers design the pricing system in order to encourage the production of pigs with special features, e.g. lean (but not too lean) pigs. The effects of such changes can be difficult to predict. The models and methods may assist in modeling changes and predict the effect of such changes on the marketing management behavior and the distribution of marketed pigs.

6.7 References


Summary

In Denmark approximately 19 million pigs are marketed for slaughtering yearly. The majority of these pigs are priced based on carcass merits and the manager of the slaughter pig operation determines when to market the individual pigs and/or groups of pigs. This problem of how to select and when to market (groups of) pigs is here defined as the “slaughter pig marketing management problem”. Most managers solve this problem by simple calculations or rules of thumb and only very few applicable and operational tools for supporting the manager in this process exits. At the same a large amount of herd–specific data are produced every day, but only few of these data are, at present, utilized in a formalized way in the management of the operation in general, and in the management of the slaughter pig marketing in particular. The objective of the research project leading to this thesis was to investigate and/or develop models and methods for handling and solving the “slaughter pig marketing management problem”, with special emphasis on utilizing the increasing (in quality and quantity) herd specific registrations. The overall aim was to develop the fundament of an applicable and operational Marketing Management Support Tool (MMST) to be used in the individual slaughter pig operation.

The MMST is represented logically and functionally by two subsystems: A general Belief Management System (BMS) used for updating the “belief in the state of the herd” based on herd specific registrations and a more specific Decision Support System (DSS) for optimizing marketing decisions. A stochastic animal growth model based on multi variate normal distributions is introduced and used as a representation of the state of the herd and as an interface between the BMS and the DSS.

The BMS is based on the EM–algorithm and other methods for reducing the bias in data caused by the selection of individual pigs for marketing and on the Kalman filter for updating the belief in the herd as new data are produced. Assuming that pigs are identified by batch–#, it is demonstrated how the BMS even in the case of very sparse data, returns fine and updated estimates of the state of the herd. One of the main features of the Kalman filter is demonstrated by examples: The quick adaption to changes in the system modeled. However, the results are based on simulated data and should not be interpreted without precautions.

“The slaughter pig marketing management problem” can be partitioned into two subproblems: (i) How to select and when to market individual pigs from batches and (ii) when
to terminate (market the remainder of) the batch and insert a new batch of weaners. The two problems can be solved independently of each other and based on basic production economics and asset replacement theory. The DSS is based on these methods, but special methods are applied in order to cope with different selection criteria and with the uncertainty and variation that exists in biological systems as the slaughter pig operation. The results show that selection on carcass leanness as well as live weight is only slightly superior to selection on live weight only and very little financial room is left for performing the on–farm leanness measuring. The selection criteria is quite unaffected by changes in model parameters (e.g. prices and growth rates), while the optimal terminal marketing stage is more affected by such changes. As is the case for the BMS, the results are based on simulated data.

Some of the optimization algorithms in the DSS are based on Recursive Dynamic Programming; an optimization method originating from an alternative approach to Dynamic Programming: Recursions. The method is interesting from a computational as well as a theoretical point of view; it is efficient and general. These features are demonstrated by examples and by the DSS.

The thesis is concluded by a general discussion of theoretical and applicable aspect of the research and the proposed models and methods, with special emphasis on the aspects of implementation as a MMST. It is concluded that the models and methods have the potential for implementation as tools for producing general guidelines and, if or when more and better registrations on individual animals are available, as MMSTs.
Sammendrag (Danish summary)


LSSS’et repræsenteres logisk og funktionelt ved to undersystemer: (i) Et generelt ForventningsStyringsSystem (FSS) som bruges til opdatering af “forventningen om bedriftens tilstand” baseret på besætningsspecifikke registreringer og et mere specifikt BeslutningsStøtteSystem (BSS) til optimering af beslutningerne vedrørende levering. En stokastisk dyrevækstmodel baseret på multivariable normalfordelinger introducerer og bruges som repræsentation af besætningsen tilstand og som et mellemled mellem FSS’et og BSS’et.

Slagtesvin–leveringsproblemet kan opdeles i to delproblemer: (i) Hvordan skal dyr udvælges fra holdet og hvornår skal de leveres og (ii) hvornår skal et hold afsluttes (de resterende svin leveres) og et nyt hold smågrise indsættes? De to problemer kan løses uafhængigt og baseret på basal produktionsøkonomi og udskiftningsteori. BSS’et er baseret på disse metoder, men specielle teknikker er anvendt for at kunne håndtere forskellige udvælgelseskriterier og usikkerheden og variationen forbundet med biologiske systemer som slagtesvinebesætningen. Resultaterne viser at udvælgelse på baggrund af både kødprocent og levendevægt kun er svagt fordelagtigt i forhold til udvælgelse på baggrund af levendevægt alene og overskuddet til betalingen af målingen af kødprocenten på bedriften er meget lille. Selektionskriterierne er relativt upåvirkede af ændringer i modelparametre (som f.eks. priser og vækstrate), mens den optimale holdlængde påvirkes mere af sådanne ændringer. Som for FSS’et gælder det at resultaterne er baseret på simulerede data.

Nogle af optimeringsalgoritmerne der anvendes i BSS’et er baseret på Rekursiv Dynamisk Programmering; en optimerings algoritme som har sit udspring i en alternativ indgangsvinkel til Dynamisk Programmering: Rekursioner. Metoden er interessant udfra en beregningsmæssig såvel som en teoretisk synsvinkel: Den er effektiv og generel. Disse egenskaber demonstreres ved eksempler og via BSS’et.

Afhandlingen afsluttes med en generel diskussion af de teoretiske og anvendelsesmæssige aspekter af forskningsarbejdet og de foreslåede modeller og metoder, med speciel vægt på implementering som et LSSS. Det konkluderes at modeller og metoder har potentielle til at kunne implementeres som værktojer til udarbejdelse af generelle retningslinjer og, hvis eller når flere og bedre registreringer på enkeltdyrsniveau bliver tilgængelige, som et LSSS.